Math 330 Section 1 - Fall 2024 - Homework 10

Published: Tuesday, October 8, 2024 Running total: 38 points
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Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1; ch.2.1 - 2.6, ch.3; skim ch.4; ch.5 - 9.5

B/G (Beck/Geoghegan) Textbook:
ch.2 – 6.3, ch.8, ch.9.2, ch.10

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, October 14:

- **a.** Carefully read MF ch.9.6. and 9.7
- **b.** Carefully read MF ch.9.8 until before Proposition 9.43 and skim the optional remainder.
- **c.** Skip the optional MF ch.9.9 (Sequences of Sets and Indicator functions and their liminf and limsup). The stronger students are encouraged to skim the contents, in particular the last remark.

Reading assignment 2 - due Wednesday, October 16:

- **a.** Carefully read B/G ch.6.4 and ch.7.1.
- **b.** Review B/G ch.8 and ch.9.

Reading assignment 3 - due Friday, October 18:

- **a.** Carefully read B/G ch.10. You have encountered the material in a more demanding setting in MF ch.9.3.
- **b.** Review B/G ch.11.1 and carefully read B/G ch.11.2.

Written assignments: See next page

Written assignment 1: Prove Proposition 7.13: Every infinite set X contains a proper subset A that is countably infinite. **Hint:** Use recursion with an induction argument to show that you can remove a_{n+1} From $X_n := X \setminus \{a_0, a_1, \dots, a_n\}$: PROVE that $X_n \neq \emptyset$. Why does that help?

Written assignment 2: Prove the following part of De Morgan's Law:

Let there be a universal set Ω which contains all elements of an indexed family of sets $(A_\alpha)_{\alpha \in I}$. Then

$$\Big(\bigcap_\alpha A_\alpha\Big)^{\complement} \subseteq \bigcup_\alpha A_\alpha^{\complement}.$$

Written assignment 3: Prove formula (8.32):

If X, Y, Z be arbitrary, nonempty sets and $f : X \to Y$, $g : Y \to Z$, $U \subseteq X$, and $W \subseteq Z$, then

$$(g \circ f)(U) \subseteq g\big(f(U)\big) \text{ for all } U \subseteq X.$$