Math 330 Section 1 - Fall 2024 - Homework 11

Published: Saturday, October 19, 2024 Running total: 40 points
Last submission: Friday, November 1, 2024

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:

ch.1; ch.2.1 - 2.6, ch.3; skim ch.4; ch.5 - 9.9

B/G (Beck/Geoghegan) Textbook:

ch.2 – 11.2

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, October 21:

- **a.** Skip the optional MF ch.9.10 (Sequences that Enumerate Parts of $\mathbb{Q}$). The stronger students are encouraged to at least skim the contents.
- **b.** Carefully read MF ch.10.1 – 10.2. Unless you are a masochist, stay away from ch.10.3.
- **c.** If you neither have taken nor are currently taking a linear algebra course, read carefully MF ch.11.1 and ch.11.2.1 through Example 11.11 (Vector spaces of real–valued functions). Otherwise, focus on the examples in MF ch.11.2.1. In particular, study the function space examples, e.g., Example 11.11 (Vector spaces of real–valued functions).

Reading assignment 2 - due: Wednesday, October 23:

- **a.** Read carefully B/G ch.12 and ch.13.1 – 13.4. You know the material from MF ch.7, 9, 10.

Reading assignment 3 - due Friday, October 25:

- **a.** Read VERY CAREFULLY MF ch.11.2.2. Skip nothing! Be sure to understand for $p = 2$ why $\|f\|_{L^p} = \left(\int_a^b |f(x)|^p dx\right)^{1/p}$ is a measure for the size of f. This will be easier if you draw a picture for $p = 1$!

Written assignment 1: Prove formula (9.14) of prop.9.11: Let X be a nonempty set and $\varphi, \psi : X \to \mathbb{R}$. Let $\emptyset \neq A \subseteq X$. Then

$$\inf\{\varphi(x) + \psi(x) : x \in A\} \geqq \inf\{\varphi(y) : y \in A\} + \inf\{\psi(z) : z \in A\}.$$

Do the proof by modifying the proof of formula (9.13). Follow that proof as closely as possible! You are **NOT ALLOWED** to apply formula (9.13) to $-\varphi$ and $-\psi$.

Written assignment 2: Prove MF prop.9.18**(b)**: If y_n is a sequence of real numbers that is non-increasing, i.e., $y_n \geqq y_{n+1}$ for all n, and bounded below, then $\lim_{n\to\infty} y_n$ exists and coincides with

$\inf\{y_n : n \in \mathbb{N}\}$. • Do the proof by modifying the proof of prop.9.18**(a)**.
You are **NOT ALLOWED** to apply prop.9.18**(a)** to the sequence $x_n := -y_n$!