Math 330 Section 1 - Fall 2024 - Homework 15

Published: Tuesday, November 13, 2024 Running total: 56 points
Last submission: Monday, December 2, 2024

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1; ch.2.1 - 2.6, ch.3; skim ch.4; ch.5 - 14.1

B/G (Beck/Geoghegan) Textbook:
ch.2 – 13.4

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, November 18:

- **a.** Read carefully MF ch.14.2 and 14.3. Understand the 3 equivalent ways to refer to compactness in metric spaces plus #4 in $\mathbb{R}^n$ Note that the "extract finite open subcovering" property needs extra careful study!

Reading assignment 2 - due: Wednesday, November 20:

- **a.** Carefully read the remainder of MF ch.14.
- **b.** Review B/G ch.13. You have encountered the material already in MF ch.7 and ch.10.

Reading assignment 3 - due Friday, November 22:

- **a.** Carefully read MF ch.15.1 and 15.3.
- **b.** Review the end of MF ch.11.2.1, starting at Definition 11.10 (Linear dependence and independence). Carefully read MF ch.15.2.

Be sure to read pages 2 – 4!

Supplementary instructions for reading MF ch.12 - 14:

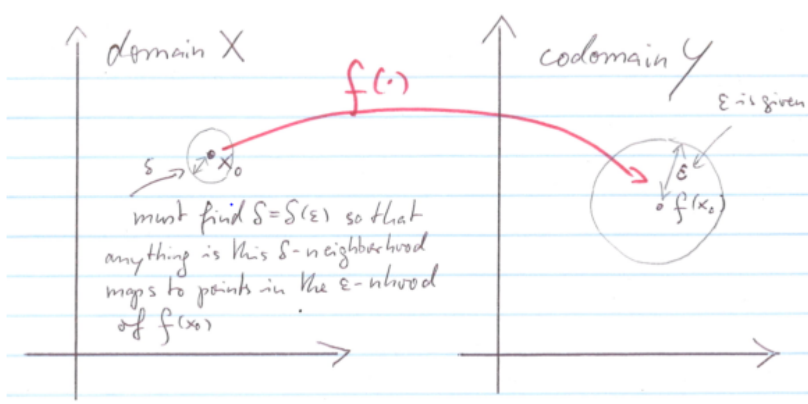
When you read or reread any topics in those chapters then the following is good advice:

a. MF ch.12.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for

- open sets and neighborhoods (ch.12.1.3)
- convergence, expressed with nhoods (the end of def.12.10 in ch.12.1.4)
- metric and topological subspaces (ch.12.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \biguplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^{\complement}$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^{\complement}$ and A_1. What is $N_\varepsilon^A(x_j)$?
- Contact points, closed sets and closures (ch.12.1.8): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize the definitions in this chapter and thm 12.6 and thm.12.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.12.2 (Continuity): Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 0.1: ε-δ continuity



Written assignments on page 3

Written assignment 1: Let X be a nonempty set. For each $j \in \mathbb{N}$ let $(x, y) \mapsto d_j(x, y)$ be a metric on X and let $a_j \in [0, \infty[$ such that $\sum_{j=1}^{\infty} a_j < \infty$ and at lease one a_j is not zero. Let

$$d(x, y) \;:=\; \sum_{j=1}^{\infty} a_j \, d_j(x, y)\,; \qquad (x, y \in X).$$

Prove that d defines a metric on X.

Written assignment 2: Prove Proposition 12.15:
Let $(X, \mathfrak{U})$ be a topological space If $A \subseteq B \subseteq X$, then $A^o \subseteq B^o$.

Written assignment 3: Let (X, d) be a metric space and $A \subseteq X$, $A \neq \emptyset$. Let

$$\gamma := \gamma(A) := \inf\{d(x, y) : x, y \in A \text{ and } x \neq y\}.$$

(a) Prove that if $\gamma > 0$ and $(x_n)_n$ is Cauchy in A, then $(x_n)_n$ is constant, eventually.
(b) Use **(a)** to prove that if $\gamma > 0$, then A is complete.
One point each for (a) and (b)!