Math 330 Section 1 - Fall 2025 - Homework 10

Published: Thursday, October 16, 2025 Running total: 37 points
Last submission: Friday, October 31, 2025

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1 - 3, skim ch.4, ch.5 - 9, ch.11 through Example 11.11

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 7, ch.8, ch.9.1, ch.10-11

Other material (optional):
B/K lecture notes ch.1.1 and ch.1.2, except ch.1.2.4
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due Monday, October 20:

- **a.** Skip the optional MF ch.9.10 (Sequences that Enumerate Parts of $\mathbb{Q}$). The stronger students are encouraged to at least skim the contents.
- **b.** Carefully read MF ch.10.1 – 10.2. Unless you are a masochist, stay away from ch.10.3.
- **c.** If you neither have taken nor are currently taking a linear algebra course, read carefully MF ch.11.1 and ch.11.2.1 through Example 11.11 (Vector spaces of real–valued functions). Otherwise, focus on the examples in MF ch.11.2.1. In particular, study the function space examples, e.g., Example 11.11 (Vector spaces of real–valued functions).

Reading assignment 2 - due: Wednesday, October 22:

- **a.** Read VERY CAREFULLY MF ch.11.2.2. Skip nothing! Be sure to understand for $p = 2$ why $\|f\|_{L^p} = \left(\int_a^b |f(x)|^p dx\right)^{1/p}$ is a measure for the size of f. This will be easier if you draw a picture for $p = 1$!

Reading assignment 3 - due Friday, October 24 (Rejuvenation day:

- **a.** Review B/G ch.9.2 and ch.8.
- **b.** Read carefully B/G ch.12 and ch.13.1 – 13.4. You know the material from MF ch.7, 9, 10.

Written assignments: See the next page!

Written assignments:

Written assignment 1: Prove Proposition 7.13: Every infinite set X contains a proper subset A that is countably infinite. **Hint:** Use recursion with an induction argument to show that you can remove a_{n+1} From $X_n := X \setminus \{a_0, a_1, \dots, a_n\}$: To be able to do so, you must PROVE that $X_n \neq \emptyset$ holds for ALL n. Why does that help? (What can you say about the sequence $(x_n)_{n=1}^{\infty}$)?

Written assignment 2: Prove the following part of De Morgan's Law:
Let there be a universal set Ω which contains all elements of an indexed family of sets $(A_\alpha)_{\alpha \in I}$. Then

$$\Big(\bigcap_{\alpha} A_\alpha\Big)^{\complement} \subseteq \bigcup_{\alpha} A_\alpha^{\complement} .$$

Written assignment 3: Prove formula (8.33):
If X, Y, Z be arbitrary, nonempty sets and $f : X \to Y$, $g : Y \to Z$, $U \subseteq X$, and $W \subseteq Z$, then

$$(g \circ f)(U) \subseteq g\big(f(U)\big) \text{ for all } U \subseteq X .$$