Math 447 - Spring 2024 - Homework 02

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

WMS (Wackerly, et al. Textbook):
Nothing assigned yet

MF447 lecture notes:
Ch.1, ch.2.1 – 2.3

Other:
Nothing assigned yet

New reading assignments:

Reading assignment 1 - due Monday, January 22:

- **a.** Carefully read MF ch.2.4. You may find it very difficult if you did not attend Math 330 Remember that I talked about the assignment $A \mapsto P(A)$ of a probability measure as a function $2^\Omega \to [0, 1]$.
- **b.** Carefully read the remainder of MF ch.2. (Two short chapters.)

Reading assignment 2 - due Wednesday, January 24:

- **a.** Carefully read MF ch.3.1. Refer back to MF ch.1 while you do it!
- **b.** Carefully read WMS ch.1. You should understand what frequency diagrams and histograms are all about

Reading assignment 3 - due Friday, January 26:

- **a.** Carefully read WMS ch.2.1 - 2.5.
- **b.** Carefully read MF ch.3.2.

Written assignments - Not collected for grading:

Remember that **some of those assignments will be relevant for the quizzes and exams.**

(a) Write rom memory the following definitions and compare them with the MF lecture notes:

- a function h with domain 2^S and codomain $\mathbb{R}$ Is $s \in S$ an arguments of h?
- Given an index set J and a family $(B_j)_{j \in J}$, what is $\bigcup\limits_{j \in J} B_j$ and what is $\bigcap\limits_{k \in J} B_k$?
- Probability space $(\Omega, \mathfrak{F}, P)$. What is $\mathfrak{F}$ called and how is it defined? Same for P.
- countable set

(b) One of the following assignments defined on the atomar events n of the sample space $\mathbb{N}$ can be extended to a probability measure on $\mathfrak{F} := 2^{\mathbb{N}}$. Which one? What is wrong with the other two?

- $\{n\} \mapsto P_1\{n\} := (1/2)^{n-1}(1/4)$
- $\{n\} \mapsto P_2\{n\} := (1/2)^{n-1}(1/2)$
- $\{n\} \mapsto P_3\{n\} := (1/2)^{n-1}(3/4)$

(c) Let $\Omega := \{1,2,3\},\ A := \{1,2\},\ B := \{2,3\}, \mathscr{A} := \{A,B\}$. The σ–algebra $\sigma\{\mathscr{A}\}$ (see Definition 3.4 and Theorem 3.3) contains 8 elements. What are they? [1]

Selected answers:

(b) Since $\mathbb{N} = \{1\} \uplus \{2\} \uplus \{3\} \uplus \cdots$, We must have $P_1(\mathbb{N}) = P_2(\mathbb{N}) = P_3(\mathbb{N}) = 1$. Let $q := 1/2$:

$$\sum_{j=0}^{\infty} q^j \frac{1}{1 - 1/2} = 2\,.$$

Thus,

$$\sum_{j=1}^{\infty} P_1\{j\} q^j = \frac{1}{4}\sum_{j=1}^{\infty} q^{j-1} = \frac{1}{4}\sum_{j=0}^{\infty} q^j = \frac{2}{4} \neq 1\,.$$

Likewise,

$$\sum_{j=1}^{\infty} P_2\{j\} q^j = \frac{1}{2}\sum_{j=1}^{\infty} q^{j-1} = \frac{2}{2} = 1\,,$$
$$\sum_{j=1}^{\infty} P_3\{j\} q^j = \frac{3}{4}\sum_{j=1}^{\infty} q^{j-1} = \frac{3 \cdot 2}{4} \neq 1\,.$$

Only P_2 can be extended to a probability measure on $2^{\mathbb{N}}$.

(c)

$$\sigma\{\mathscr{A}\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\,.$$

[1] Definition 3.4 and Theorem 3.3 have been marked with $\boxed{\star}$ as optional, so you are not asked to write them down from memory. If a quiz or exam is about such an optional item then the definition or statement will be given to you.