Math 447 - Spring 2024 - Homework 03

Published: Tuesday, January 23, 2024

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

WMS (Wackerly, et al. Textbook):

ch.1 - 2.5

MF447 lecture notes:

Ch.1 - 2, ch.3.1 - 3.2

Other:

Nothing assigned yet

New reading assignments:

Reading assignment 1 - due Monday, January 29:

- **a.** Carefully read the remainder of MF ch.3. I already showed you at the end of lecture #2, without using the terminology, how the preimage of the random variable $Y : (i, j) \mapsto i+j$ transports the probability $P\{(i, j)\} = \frac{1}{36}$ associated with the roll of two dice into $P_Y\{k\}$ = probability that the sum of the rolls is k = $P\{Y = k\}$
- **b.** Carefully read WMS ch.2.6 through Example 2.10.

Reading assignment 2 - due Wednesday, January 31:

- **a.** Carefully read the remainder of WMS ch.2.6.
- **b.** Carefully read MF ch.4.

Reading assignment 3 - due Friday, February 2:

- **a.** Carefully read the remainder of WMS ch.2.
- **b.** Carefully read MF ch.5.

Written assignments - Not collected for grading:

Remember that **some of those assignments will be relevant for the quizzes and exams.**

(a) Write rom memory the following definitions and compare them with the MF lecture notes:

- preimage of an event $U \subseteq ??$, given a function $f : X \to Y$; preimage as a function $f^{-1} : ?? \to ??$ which makes the assignment $?? \mapsto ??$
- Given a random variable $Y : (\Omega, P) \to \mathbb{R}$ and an interval $[a, b] \subseteq \mathbb{R}$, write $P_Y([a, b]) = P\{??\}$.
- countable set
- discrete probability space

(b) Given is a probability space $(S, \mathscr{S}, P)$ with events $A, B, C, D, E \in \mathscr{S}$.

(1) If $P(A) = 0.5, P(B) = 0.4, P(A \cap B) = 0.2$, what is $P\big((A \cup B)^{\complement}\big)$?

(2) If $P(C \triangle D) = 0.6, P(E) = 0.3, P(C \cap D) = 0.2$, what is $P\big(C^{\complement} \cap D^{\complement}\big)$?

(c) Let $X := \{0,1\}^3 = \{(x_1, x_2, x_3) : x_j = 0 \text{ or } x_j = 1 \text{ for each } j = 1, 2, 3\}$.
Let the function h be $h : X \to [-2.5, 2\pi[\,;\ (x_1, x_2, x_3) \mapsto x_1 + x_2 + x_3$.

(1) What is $h^{-1}([-1.5, 1.5]])$?

(2) What is $\{h > 2.71828\}$?

(d) Do closed book the fully worked exercises of WMS ch.2.4 and 2.5

(e) All WMS exercises below are odd-numbered, so the solutions are in the book.

- WMS ch.2.5 exercises: #2.27, 2.29, 2.33
- WMS ch.2.6 exercises: #2.35, 2.37, 2.43, 2.45, 2.55, 2.61, 2.68
- WMS ch.2.7 exercises: #2.71, 2.75, 2.79
- WMS ch.2.8 exercises: #2.95, 2.101, 2.107

MF note to #2.45: The solutions manual lists $\binom{17}{2,7,10} = 408,408$ as answer. I did not check whether $408,408$ is correct, but the multinomial coefficient is not: It should be $\binom{17}{2,5,10}$.

Selected answers:

Solution to (b.1):

$$\begin{aligned} \Omega &= A \setminus B \uplus A \cap B \uplus B \setminus A \uplus (A \cup B)^{\complement} \\ \Rightarrow\ P(\Omega) &= P\big(A \setminus B\big) + P\big(A \cap B\big) + P\big(B \setminus A\big) + P\big((A \cup B)^{\complement}\big) \\ \Rightarrow\ 1 &= 0.3 + 0.2 + 0.2 + P\big((A \cup B)^{\complement}\big) \ \Rightarrow\ \boxed{P\big((A \cup B)^{\complement}\big) = 0.3} \ \blacksquare \end{aligned}$$

Solution to (b.2):

$$\begin{aligned} (C \cup D) = (C \triangle D) \uplus (C \cap D) &\Rightarrow P(C \cup D) = 0.6 + 0.2 = 0.8 \\ &\Rightarrow P\big(C^{\complement} \cap D^{\complement}\big) = P\big((C \cup D)^{\complement}\big) = 1 - 0.8 = \boxed{0.2} \ \blacksquare \end{aligned}$$

Solution to (c1):

$$\begin{aligned} h(x_1, x_2, x_3) \in [-1.5, 1.5] &\Leftrightarrow h(x_1, x_2, x_3) = 0 \text{ **or** } h(x_1, x_2, x_3) = 1 \\ &\Leftrightarrow (x_1, x_2, x_3) \text{ is one of } (0,0,0),\ (0,0,1),\ (0,1,0),\ (1,0,0) \\ &\Rightarrow h^{-1}([-1.5, 1.5]) = \{(0,0,0),\ (0,0,1),\ (0,1,0),\ (1,0,0)\} \ \blacksquare \end{aligned}$$

Solution to (c2):

$$\begin{aligned} h(x_1, x_2, x_3) > 2.71828 &\Leftrightarrow h(x_1, x_2, x_3) = 3 \Leftrightarrow (x_1, x_2, x_3) = (1,1,1) \\ &\Rightarrow \{h > 2.71828\} = \{(1,1,1)\} \ \blacksquare \end{aligned}$$