Math 447 - Spring 2024 - Homework 12

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Status - Reading Assignments:

Here are the reading assignments you were asked to complete before the first one of this HW.

WMS (Wackerly, et al. Textbook): ch.1 - 6

MF447 lecture notes: Ch.1 - 10.1

Other: Nothing assigned yet

New reading assignments:

- When studying the WMS book, be sure to pay extra attention to the examples!

Reading assignment 1 - due Monday, April 8:

a. Carefully read MF ch.10.2.
b. Carefully read MF ch.10.3 through Theorem 10.8.

Reading assignment 2 - due Wednesday, April 10:

a. Carefully read the remainder of MF ch.10.3.
b. Carefully read WMS ch.7.1(!) – 7.2.

Reading assignment 3 - due Friday, April 12:

a. Carefully read MF ch.10.4.
b. Carefully read WMS ch.7.3.

Written assignments - Not collected for grading:

- When studying the WMS book, be sure to pay extra attention to the examples!

(a) Write from memory the following definitions and compare them with the MF lecture notes:

- The formula to compute f_U from f_Y for $U = h(Y)$ (1-dim case)
  What properties must h have so you can apply it?
- The formula to compute $f_{\vec{U}}$ from $f_{\vec{Y}}$ for $\vec{U} = \vec{h}(\vec{Y})$ (n-dim case)
  What properties must $\vec{h}$ have so you can apply it?
- How is $Y_{(4)}$ defined?
- Can you derive the formulas $F_{Y_{(1)}(y)} = 1 - [1 - F(y)]^n$ and $F_{Y_{(n)}(y)} = [F(y)]^n$?
- Can you derive $f_{Y_{(1)}(y)} = n \cdot [1 - F(y)]^{n-1} \cdot f(y)$ and $f_{Y_{(n)}(y)} = n \cdot [F(y)]^{n-1} \cdot f(y)$?
- Apply combinatorics (permutations) to order statistics: Theorem 9.4 (Joint PDF of the order statistic) + subsequent examples.
- Can you derive the formulas $F_{Y_{(1)}(y)} = 1 - [1 - F(y)]^n$ and $F_{Y_{(n)}(y)} = [F(y)]^n$?
- Can you derive $f_{Y_{(1)}(y)} = n \cdot [1 - F(y)]^{n-1} \cdot f(y)$ and $f_{Y_{(n)}(y)} = n \cdot [F(y)]^{n-1} \cdot f(y)$?

(b) Some of the following will be answered below.

1. • If Y_1, Y_2 are independent and discrete, what is the relationship between $p_{Y_1|Y_2}(y_1 \mid y_2)$ and $p_{Y_1(y_1)}$? • If Y_1, Y_2 are independent and continuous, what is the relationship between $f_{Y_1|Y_2}(y_1 \mid y_2)$ and $f_{Y_1(y_1)}$?

2. If U, V are independent, • what is $E[3V^2 \mid U = 5]$ Prove your result for discrete U and V.
• what is $Var[U \mid V = v_0]$ Prove your result for continuous U and V.
• what are $E[3V^2 \mid U]$ and $Var[U \mid V]$ U and V may be continuous or discrete.

(c) All WMS exercises below are odd-numbered, so the solutions are in the book.

- WMS ch.6.6 exercises (bivariate transforms): #6.65, 6.69,
- WMS ch.6.5 exercises (MGF method): #6.37, 6.39, 6.41, 6.43
- WMS ch.6.7 exercises (order statistics): #6.73, 6.81, 6.85
- WMS ch.7.2 exercises (Sampling distributions related to the normal distribution): #7.9, 7.11, 7.15

(d) Some solutions for **(b):**

(Solution for (b.1): If Y_1, Y_2 are independent and discrete, then

$$p_{Y_1,Y_2}(y_1, y_2) \;=\; p_{Y_1}(y_1) \cdot p_{Y_2}(y_2) \;\;\Rightarrow\;\; p_{Y_1|Y_2}(y_1 \mid y_2) \;=\; p_{Y_1}(y_1)$$

If Y_1, Y_2 are independent and continuous, then

$$f_{Y_1,Y_2}(y_1, y_2) \;=\; f_{Y_1}(y_1) \cdot f_{Y_2}(y_2) \;\;\Rightarrow\;\; f_{Y_1|Y_2}(y_1 \mid y_2) \;=\; f_{Y_1}(y_1)$$

(Solution for (b.2): It follows from **(b.1)** that conditioning has no effect. Thus,

• $E[3V^2 \mid U = 5] = E[3V^2]$ • $Var[U \mid V = v_0] = Var[U]$
• $E[3V^2 \mid U] = E[3V^2]$ and $Var[U \mid V] = Var[U]$