

MATH447
Fall 2020
Practice Midterm 1 (Edition 1)

1. Three cards are withdrawn from a standard 52 card deck without replacement.

- (a) What is the probability of getting two distinct suits?
- (b) What is the probability of getting all three cards of the same rank?

2. The Lakers and Heat are playing in the NBA Finals. The series is a best-of-seven (first team to win four games clinches the series). The Lakers will win each game with probability $3/4$.

- (a) Given that the Heat won game one, what is the probability the Lakers go on to win the series?
- (b) Given that the Heat win at least two games in the series, what is the probability the Lakers go on to win the series?

3. 10 people consist of five men and five women. Four people are selected from this group of 10 to form a committee.

- (a) What is the expected number of men in the committee?
- (b) What is the probability that the number of men in the committee is unequal to the number of women in the committee?
- (c) Given that the number of men and women are unequal in the committee, what is the probability that the committee either consists of all men or all women?

4. A multiple choice test consists of 25 questions. For each question, there are five choices, but only one correct answer. A student has not studied for this test and will guess each question randomly.

- (a) Use Chebyshev's Inequality to give a lower bound on the probability that the number of questions answered correctly is between 2 and 8, inclusive.
- (b) Compute the probability that the student answers no more than one question correctly.

5. A coin is tossed until a head appears. The probability that the coin shows heads is $1/1000$, and let X be the number of coin tosses. Let $\mu = E(X)$ and

$$\sigma^2 = \text{Var}(X).$$

- (a) Compute μ and σ^2 .
- (b) Compute $P(X \geq 1098)$.

6. A random variable X has the moment generating function (mgf)

$$M_X(t) = c \sum_{k=0}^4 e^{k^2 t}$$

where $c > 0$.

- (a) Determine c .
- (b) Compute $E(\sqrt{X})$.
- (c) Compute $Var(\sin(\frac{\pi}{2}X))$.