Math 454 - Spring 2021 - Homework 06

Published: Friday, February 19, 2021

Update March 22, 2021

Added written assignments.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date plus the material taught in class without assigned reading.

SCF2 (Shreve Textbook):
ch.1-4.2

MF454 lecture notes:
ch.2 – 6, ch.7 (all that is published so far)

Other: • Wiersema (optional - no internet PDF) p.130-135: one period binomial model

New reading assignments:

Reading assignment 1 - due Monday, March 22:

a. Carefully read SCF2 ch.4.4.1. The proof of thm.4.4.1 is not as important as remark 4.4.2 that follows it.

Reading assignment 2 - due: Wednesday, March 24:

b. Carefully read SCF2 ch.4.4.2. Thm.4.4.6 (Itô formula for Itô processes) will be the most important tool for continuous time finance.

Reading assignment 3 - due Friday, March 26:

a. Carefully read SCF ch.4.4.3. Dig into the technical aspects of those examples!

b. Carefully read SCF ch.4.5.1 and ch.4.5.2. Make a connection to the binomial model: What we called there V_t^H he calls X_t, what we called $\Pi_(\mathcal{X})$ he calls $c(t, S_t)$.

Written assignments: on page 2.

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

Practice pricing options in the binomial multiperiod model. You will probably need a calculator or a spreadsheet to do them. Follow Example 6.1 in MF454.

Here are some suggestions.

(1) European call with $T = 2, K = 50, R = 0, u = 1.2, d = 0.8, p_u = 0.7, s = S_0 = 60$,
(2) Same as **(1)**, but $K = 60$.
(3) Same as **(1)**, but $K = 70$.
(4) Same as **(1)**, but $K = 40$.
Does it make sense to you how $\Pi_0(\mathcal{X})$ and strike price are related?

Note that the probabilities p_u, p_d of the real world probability P are not part of those computations. But you should use it to compare the correct price $\Pi_0(\mathcal{X}) = \frac{1}{(1+R)^T} E^Q[\Phi(S_T)]$ with the real world expectation $\frac{1}{(1+R)^T} E^P[\Phi(S_T)]$ of the discounted contract value.

(5) Do **(1)–(4)** for a European put instead of a European call.
A **European put** option is a contract written at some time t_0. It specifies that at the time of expiration $T > t_0$ the holder of this option has the right, but not the obligation, to sell a share of an underlying security for the price of K (strike price). Note that the contract function which specifies the value of this derivative at time T to the contract holder is

$$\Psi\big(S_T(\omega)\big), \quad \text{where } \Psi(x) = (K - x)^+ = \max(K - x, 0).$$

(6) Do **(1)–(4)** with $R = 0.1$ instead of $R = 0$. Careful: You no longer have $q_d = q_u = \frac{1}{2}$!
Solutions: Will not be given, but you can verify the correctness of your computations as follows.

- Once you have computed all options prices $\Pi\big(\mathfrak{N}_{t,k}\big)$ go forward in the tree and compute the replicating portfolio holdtings (x_{t+1}, y_{t+1}) for each node.

(2) Compute the portfolio value based on those quantities (and stock price and interest rate) for each node. It should match the options price.
(3) If you have a mismatch: Remember that the holdings x_{t+1} and y_{t+1} were purchased at time t but they determine V_{t+1} at time $t + 1$, so you must multiply y_{t+1} with S_{t+1} and not with S_t.