Math 454 - Spring 2021 - Homework 07

Published: Saturday, February 27, 2021

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date plus the material taught in class without assigned reading.

SCF2 (Shreve Textbook):
ch.1-4.2, 4.4, 4.5.1, 4.5.2.

MF454 lecture notes:
ch.2 – 6, ch.7.1-7.3

Other: • Wiersema (optional - no internet PDF) p.130-135: one period binomial model

New reading assignments:

Reading assignment 1 - due Monday, March 29:

- **a.** Carefully read the MF454 additions to ch.7 in ver 2021-03-25 Skim the additions in ch.7.4.
- **b.** Carefully read SCF2 ch.4.3. I forgot to ask for that last week.

Reading assignment 2 - due: Wednesday, March 31:

- **a.** Carefully read SCF2 ch.4.4.1 and 4.4.2. **The Itö formula and understanding how to work with it is the centerpiece of this course!**

Reading assignment 3 - due Friday, April 2:

- **a.** Carefully read the rmainder of SCF2 ch.4.4. Focus on how the Itö formula is used in the examples of ch.4.4.3!

Written assignments: on page 2.

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

Written assignment 1:
What I saw in quiz 4 makes me doubt that more than just very few of you attempted to do the homework in ch.6. It is very likely that I will a problem of that nature on the second midterm, so go back to homework 6 do at least one problem with interest rate zero and one with a nonzero interest rate.

Written assignment 2:
Do SCF2 Exercise 4.1. In it Brownian motion W_t is replaced with a martingale M_t when defining the integral $\int_0^t Z_u\,dM_u$ for a simple process $Z = Z_t$:

$$\int_0^t Z_u\,dM_u \;:=\; \sum_{j=0}^{k-1} Z_{t_j}[M_{t_{j+1}} \,-\, M_{t_j}] + \Delta(t_k)[M_t(\omega) \,-\, M_{t_k}(\omega)].$$

You are asked to prove that $I_t := \int_0^t Z_u dM_u$ is a martingale.

Hint: The proof is literally that given in SCF2 Theorem 4.2.1 with one exception. You no longer have $\mathfrak{F}_t$–independence of the increments $M_u - M_{u'}$ for $t \leqq u' < u$ and must work directly with the martingale property of M_t

Written assignment 3:
If $(u,\omega) \mapsto Z_u(\omega)$ is a simple process then the Itö integral $\int_0^t Z_u\,dW_u$ was defined trajectory by trajectory, i.e., ω by ω, as

$$\int_0^t Z_u(\omega)\,dW_u(\omega) \;=\; \sum_{j=0}^{k-1} Z_{t_j}(\omega)[W_{t_{j+1}}(\omega) \,-\, W_{t_j}(\omega)] + \Delta(t_k)[W_t(\omega) \,-\, W_{t_k}(\omega)].$$

See SCF2 formula (4.2.2). So no probabilities are necessary in the above. Make a connection between $\int_0^t Z_u(\omega)\,dW_u(\omega)$ and the expression $\int f dg$ which you have seen, e.g., in the integration by parts formula

$$\int f\,dg \;=\; fg \,-\, \int g\,df.$$