Math 454 - Spring 2021 - Homework 09

Published: Friday, April 9, 2021

Update April 15, 2021
There still was a mistake. Everything in the big yellow box was CORRECT. There I stated that »It does not matter how the stock price became 85 *at* $t = 5$*.« I forgot to update the instructions I gave in assignment 1 where I stated INCORRECTLY that stock price »at time of expiration« is* 85*.*
It should be instead »at ***time of evaluation****, i.e.,* $t = 5$*.« I have corrected assignment 1 accordingly.*
Update April 14, 2021
Written assignments were added. They could be important for quiz and midterm! This version was corrected at 6:50 PM on Wed 4/14.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date plus the material taught in class without assigned reading.

SCF2 (Shreve Textbook):
ch.1-4.5

MF454 lecture notes:
ch.2 – 8

Other:
- Wiersema (optional - no internet PDF) p.130-135: one period binomial model

New reading assignments:

Reading assignment 1 - due Monday, April 12:

a. As soon as I publish the next edition: Carefully read whatever it may contain in a new ch.9 (Black–Scholes Model Part I: The PDE). I also plan to make some changes to ch.6, mostly to ch.6.1 (Basic Definitions about Financial Markets).
b. Carefully read SCF2 ch.4.6.1. It is very brief.

Reading assignment 2 - due: Tuesday, April 8:

a. Carefully read the remainder of SCF2 ch.4.6. Note that we will skip the remainder of SCF2 ch.4.

Reading assignment 3 - due: Wednesday, April 7:

a. Carefully read SCF2 ch.5.1 and ch.5.2.1.
b. Review MF ch.4.6 while you study SCF2 ch.5.2.1.

Reading assignment 4 - due Friday, April 9:

a. Carefully read SCF2 ch.5.2.2 – ch.5.2.4.

Written assignments: See next page.

Written assignments:

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

Written assignment 1:
Given is a Black–Scholes market with $r = 0.2, \alpha = 0.1, \sigma = 0.4, S_0 = 80$. You today ($t = 0$) sold a European call option based on the stock with an expiration date of $T = 10$ at a strike price of $K = 90$. You build a hedging portfolio for that option. How much of its value will be invested in the stock at time $t = 5$ if its price **AT THIS TIME**, $t = 5$, is 85?

What about various other times and stock prices?

Hint: Your answer to your prayers can be found in the first 20 lines or so of SCF2 Chapter 4.5.5 (The Greeks). Also note that not all the info I gave is needed to compute the answer.

How to choose the correct parameters for the software:
It will follow in a later chapter from the general theory that stock price S_t is a Markov process and thus conditioning on the present stock price S_t is the same as conditioning on the the entire past $\mathfrak{F}_t$. It does not matter how the stock price became 85 at $t = 5$. It is as if the clock is reset to zero at $t = 0$, as if S_0 were 85, and as if time of expiration were $T - 5 = 5$.
So you plug the following into your program:
$r = 0.2, \sigma = 0.4, S_0 = 85$ (NOT 80!), $T = 5$, $K = 90$.
Note that α does not appear in the computation of $c(t, x)$ and can be ignored.

Written assignment 12
Practice pricing options in the Black Scholes market model. You will need appropriate software to do that. Here are two resources that I have come across and rather like.

a. Magnimetrics Excel implementation:
`https://magnimetrics.com/black-scholes-model-first-steps/`.
b. Drexel U Finance calculator:
`https://www.math.drexel.edu/~pg/fin/VanillaCalculator.html`.

I am **seriously considering** putting a problem on quiz 7 or the second midterm or both asking you to use the Drexel U software (**specifically that one!**) to do some finance calculation, so be sure to practice!

- Note that there is no provision for the instantaneous mean rate of return α since it disappears in the arbitrage free pricing function of the call.
- You will find an additional parameter, usually named q, for the dividend rate. We have not yet discussed dividends yet, so $q = 0$, but you are encouraged to play around with nonzero dividend rates.
- Those of you who have a background in the R programming language, maybe because you took Math 329, are encouraged to look for R–based solutions. I am sure they exist.
- A final suggestion: Use two different resources and cross–check the results. If they match then chances are that you understand how to use this kind of software.