

## Math 454 - Spring 2023 - Homework 02

*Published: Wednesday, January 25, 2023*

### Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

SCF2 (Shreve Textbook):

ch.1.1-1.5

MF454 lecture notes:

ch.2 - 3

Other:

Nothing assigned yet

### New reading assignments:

#### Reading assignment 1 - due Wednesday, January 25:

- a. If you noticed this in time, carefully read MF ch.4 from Definition 4.5 through the end of ch.4.2. Do not (yet) worry about the proofs, not even the ones not marked as optional. Rather, try to make sense of the remarks and examples!

#### Reading assignment 2 - due Friday, January 27:

- a. Carefully read MF ch.4.3. Do not skip the remarks and examples!

#### Reading assignment 3 - due Monday, January 30:

- a. Carefully review MF ch.4 through ch.4.3. This time also look at the non-optional proofs.
- b. Carefully read MF ch.4.4.

#### Reading assignment 4 - due: Wednesday, February 1:

- a. Carefully read MF ch.4.5 – 4.6.
- b. Carefully read SCF2 ch.1.6.

#### Reading assignment 5 - due Friday, February 3:

- a. Carefully read MF ch.4.7 and 4.9.
- b. Skim MF ch.4.8.

Written assignments are on the next page.

## Written assignments:

**General note on written assignments:** I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

### Written assignment 1:

- a. Do a proof “by picture” of Proposition 2.1.: Draw the Venn diagrams and convince yourself that they match.
- b. Do the same for Proposition 2.4 **b–h**.

### Written assignment 2:

- a. Prove closed book formula (2.13).a:  $(A \cup B)^c = A^c \cap B^c$ .
- b. Theorem 3.1 has the general formula for arbitrary families: Prove that  $(\bigcup_{\alpha} A_{\alpha})^c = \bigcap_{\alpha} A_{\alpha}^c$ .

### Written assignment 3:

Remark 2.8 states that  $x = x^+ - x^-$ ,  $|x| = x^+ + x^-$ ,  $f = f^+ - f^-$ ,  $|f| = f^+ + f^-$ . Prove it.

### Written assignment 4:

- a. Write the function  $f : -1, 2[ \rightarrow \mathbb{R}$ ; as a family.
- b. Write the family  $([x, 2x])_{x \in \mathbb{Q}}$  as a function. In particular, what are domain and codomain?

### Written assignment 5:

- a. Pick at least two parts each from Examples 3.4–3.7 and verify the answers.
- b. Prove closed book formula (3.16): Assume that  $X, Y$  be nonempty,  $f : X \rightarrow Y$ ,  $J$  is an arbitrary index set,  $B \subseteq Y$ ,  $B_j \subseteq Y$  for all  $j$ . Then  $f^{-1}(\bigcap_{j \in J} B_j) = \bigcap_{j \in J} f^{-1}(B_j)$

For the answer, see ch.8 of my Math 330 lecture notes for either Fall 2020 or Fall 2022.

### Written assignment 6:

- a. Do closed book the proof of Proposition 4.3 If  $\mathfrak{E}_1 \subseteq \mathfrak{E}_2$ , then  $\sigma(\mathfrak{E}_1) \subseteq \sigma(\mathfrak{E}_2)$ .
- b. Do closed book the proof of Proposition 4.4 If  $\mathfrak{E}_1 \subseteq \sigma(\mathfrak{E}_2)$ , and  $\mathfrak{E}_1 \subseteq \sigma(\mathfrak{E}_1)$ , then  $\sigma(\mathfrak{E}_1) = \sigma(\mathfrak{E}_2)$ .