Math 454 - Spring 2023 - Homework 03

Published: Wednesday, February 1, 2023

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

SCF2 (Shreve Textbook):
ch.1

MF454 lecture notes:
ch.2 - 4.9

Other:
Nothing assigned yet

New reading assignments:

Reading assignment 1 - due Monday, February 6:

- **a.** Carefully read MF ch.5.1. With the exception of Remark 5.1, all material is marked optional, thus it will not appear on your quizzes and exams. Nevertheless, I strongly urge you to study the propositions and theorems, since I will cite them many times. Note that you might be tested on Remark 5.1.
- **b.** Carefully read MF ch.5.2. Pay special attention to the lengthy introduction! Review your Math 447 book if this introduction does not make sense!

Reading assignment 2 - due: Wednesday, February 8:

- **a.** Carefully read SCF2 ch.2.

Reading assignment 3 - due Friday, February 10:

- **a.** Carefully read the remainder of MF ch.5 (ch.5.3). You have read twice about conditional expectations. It is crucial that you reflect on this material, since it is the foundation for martingales as the best estimate of the future development of a stochastic process if no systematic trend is involved.
- **b.** Carefully read MF ch.6.1 through Remark 6.2.

Written assignments are on the next page.

Written assignments:

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

Note that some of the open book problem proofs were hidden in versions earlier than 2023-0201!

Written assignment 1:

Prove closed book Theorem 4.1: Let $f : (\Omega, \mathfrak{F})$ and $(\Omega', \mathfrak{F}')$ and $\mathfrak{E}' \subseteq \mathfrak{F}'$ such that $\sigma(\mathfrak{E}') = \mathfrak{F}'$. Then,

$f^{-1}(A') \subseteq \mathfrak{F}$ for all$A' \in \mathfrak{E}' \Rightarrow f \in m(\mathfrak{F}, \mathfrak{F}')$.

Written assignment 2:

a. In the proof of Proposition 4.11, the assertion $f(\omega) < g(\omega) \Leftrightarrow$ there is (at least one) $q \in \mathbb{Q}$ such that $f(\omega) < q < g(\omega)$ is made. Prove it!
Hint: • For any real numbers $\alpha < \beta$ there exists $q \in \mathbb{Q}$ such that $\alpha < q < \beta$ • $\mathbb{Q}$ is countable.

b. Prove closed book Proposition 4.12: The image measure μ_f is a measure on $\mathfrak{F}'$.

Written assignment 3:
Prove closed book Proposition 4.14: Every process X_t is adapted to its own filtration

Written assignment 4: Prove closed book Proposition 4.15: If τ is a random time on a filtered probability space $(\Omega, \mathfrak{F}, (\mathfrak{F}_t)_t)$, then

$$\tau \text{ is a stopping time} \quad \Leftrightarrow \quad \text{the process } (t, \omega) \mapsto X(t, \omega) := 1_{[0,\tau(\omega)[}(t) \text{ is } \mathfrak{F}_t\text{–adapted.}$$