Math 454 - Spring 2023 - Homework 09

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

SCF2 (Shreve Textbook):
ch.1 - 4 (skip ch.4.7)

MF454 lecture notes:
ch.2 - 12.6 through Remark 12.9

Other:
Nothing assigned yet

New reading assignments:

Reading assignment 1 - due Monday, March 20:

- Carefully read the remainder of MF ch.12.

Reading assignment 2 - due: Wednesday, March 22:

- Carefully read SCF2 ch.5.1 anc ch.5.2.1 - 5.2.2.

Reading assignment 3 - due Friday, March 24:

(a) Carefully read SCF2 ch.5.2.3 - 5.2.4.
(b) Read SCF2 ch.5.2.5. Skim the proofs. the better students should try to understand how the Independence Lemma (MF Theorem 5.7 and SCF2 Lemma 2.3.4) is used.

Prove the following assignments closed book (once you have looked up the proposition).

Written assignment 1: Write from memory the definition of a d–dimensional brownian motion, the corresponding multiplication table for the differentials, and the Itô formula (10.8) for a function $f(t, x, y)$, where the Itô processes are driven by a twodimensional Brownian motion.

Be sure to also write from memory the differentials of those Itô processes. It does not matter that you use matching symbols, but they must be consistent!

Would you be able to repeat this exercise with a fourdimensional Brownian motion?

Written assignment 2: Use the multidimensional Itô formula and brute force to prove Proposition 10.1.

Written assignment 3: Write from memory Lévy's characterization of Brownian Motion first for dimension one, then for the multidimensional case.

Written assignment 4: Work line by line through the (optional) proof of Proposition 10.2. It is perfect to practice multidimensional Itô calculus.

Written assignment 5: Do the proof of formula (11.1) of Proposition 11.1. Repeat this exercise every three days until you can do this efficiently.

Written assignment 6: Remember this part about Girsanov: If an adapted process Θ_t satisfies some integrability condition, then one can construct a probability $\widetilde{P}$ equivalent to P such that $d\widetilde{W_t} = \Theta_t dt + dW_t$ becomes a Brownian motion for $\widetilde{P}$. It's $d\widetilde{\vec{W}_t} = \vec{\Theta}_t dt + d\vec{W}_t$ in the multidimensional case.