Math 454 - Spring 2023 - Homework 10

Published: Friday, March 24, 2023

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

SCF2 (Shreve Textbook):
ch.1 - 5.2.5

MF454 lecture notes:
ch.2 - 12

Other:
Nothing assigned yet

New reading assignments:

Reading assignment 1 - due Monday, March 27:

- Review SCF2 ch.5.3 and read carefully SCF2 ch. 5.4. You have encountered the material in the MF lecture notes.

Reading assignment 2 - due: Wednesday, March 29:

- Carefully read MF ch.13.1 - 13.2

Reading assignment 3 - due Friday, March 31:

- Carefully read the remainder of MF ch.13.

A written assignment is on the next page.

Written assignment 1: You want to hedge a contingent claim $\mathcal{X}$ in a classical Black–Scholes market. The parameters are:

$$r = 0.05, \quad \alpha = 0.2, \quad \sigma = 2.0, \quad T = 20\,.$$

You ask your favorite psychic for advice. She tells you that not everything is revealed to her, but she can tell you that the hedge you want to set up now, at $t = 0$, must have a portfolio value of $V_0 = 80$.

Can you find out from the data given to you the expected value $\widetilde{E}[\mathcal{X}]$ of the claim at time of expiration under risk–neutral measure $\widetilde{P}$?

What about $\widetilde{E}\big[\Pi_{10}(\mathcal{X})\big]$?

Solution: First, note that $D_t = e^{-rt}$ for $0 \leqq t \leqq T$. In particular, $D_0 = 1$.

Since the hedge is self–financing and $\widetilde{P}$ is a martingale measure, discounted portfolio value $D_t V_t$ is a $\widetilde{P}$–martingale. Since $\Pi_t(\mathcal{X}) = V_t$ by the pricing principle, $D_t \Pi_t(\mathcal{X}) = D_t V_t$ for $0 \leqq t \leqq T$. It follows that

$$80 \;=\; D_0 V_0 \;=\; \widetilde{E}[D_0 V_0] \;=\; \widetilde{E}[D_t V_t] \;=\; e^{-rt}\, \widetilde{E}[V_t]\,,$$

hence,

$$\widetilde{E}[V_t] \;=\; 80 \cdot e^{rt} \;=\; 80 \cdot e^{t/20}\,, \quad \text{for } 0 \leqq t \leqq T\,.$$

Now it is easy to answer both questions.

(a) $\widetilde{E}[\mathcal{X}] \;=\; \widetilde{E}[V_T] \;=\; \widetilde{E}[V_{20}] \;=\; 80 \cdot e^{20/20} \;=\; 80\, e.$
(b) $\widetilde{E}\big[\Pi_{10}(\mathcal{X})\big] \;=\; \widetilde{E}[V_{10}] \;=\; 80 \cdot e^{10/20} \;=\; 80 \cdot e^{1/2}.$