Math 488P/588 Homework #3

Due Thursday, 2/5/2015 NAME: ______________________________

Assignment 1: Read Section Bluman Section 4.1

Assignment 2: Bluman Problems P. 206/209: #12, #6, #18, #27, #44, #47

Assignment 3: Prove the equality of the following two formulas for the sample variance (Bluman p. 141, 142):

$$\sigma^2 = \frac{\sum_{j=1}^{n}\left(X_j - \bar{X}\right)^2}{n-1} = \frac{n\sum_{j=1}^{n} X_j^2 - \left(\sum_{j=1}^{n} X_j\right)^2}{n(n-1)}$$

Assignment 4: Read the Hossain/Makhnin lecture notes until the start of Ch.3 on p. 45. If that feels like too much: read through the end of Ch.2.4: It might save you time on the following assignments. If you have never before seen the basic counting rules or the notion of conditional probability you should look at Ch.2.5 and also 2.6 before tomorrow's lecture or you might feel overwhelmed when I go through those topics at high speed.

For assignment 5, here is a reminder about families of sets:

Definition Hwk03-1:
Let J (the "index set") and Ω (the "universal set") be arbitrary non-empty sets and assume that with each $j \in J$ we associated a set $A_j \subset \Omega$. We write

$$\left(A_j\right)_{j\in J} \text{ or } \left(A_j\right)_j \text{ or } \left(A_j\right)$$

and call this a collection of sets indexed by J.

You have seen some examples in class or other math courses:

Vectors $(X_j)_{1\leq j\leq n}$ of (real valued) random variables where
$X_j : (\Omega, \ , \ ) \longrightarrow \mathbb{R}$ and $J = \{1, 2, \dots, n\}$

Sequences $(a_j)_{j\in\mathbb{N}}$ of numbers a_j and sequences $(A_j)_{j\in\mathbb{N}}$ of subsets $A_j \subseteq \Omega$.

Definition Hwk03-2: Arbitrary intersecions and unions of families of sets $(A_i)_{i\in J}$:

$$x \in \bigcap_i A_i \iff x \in A_i \ \forall i \in J \quad \text{and} \quad x \in \bigcup_i A_i \iff x \in A_i \text{ for at least one } i \in J$$

Finally a reminder that the most common method of proving that two sets P and Q are equal is to show that

$$\text{both} \quad P \subseteq Q \text{ (i.e., } x \in P \Rightarrow x \in Q) \quad \text{and} \quad Q \subseteq P \text{ (i.e., } x \in Q \Rightarrow x \in P)$$

Assignment 5:

Prove the general laws of distributivity for sets $A, B_j \subseteq \Omega$ ($j \in J$):

$$\textbf{a.} \quad A \cup \bigcap_j B_j = \bigcap_j (A \cup B_j)$$

$$\textbf{b.} \quad A \cap \bigcup_j B_j = \bigcup_j (A \cap B_j)$$

If you have problems with arbitrary index sets, prove this equality for sequences (i.e., $j = \mathbb{N}$. The proof is no different!

Assignment 6:

Use (one of) the laws of distributivity for sequences of sets to show the following: Let $(\Omega, ??, P)$ be a probability space, i.e., P is a probability on Ω and let $A \subseteq \Omega$ such that $P(A) > 0$. In case you forgot: you'll find the probability axioms before the start of Ch.2.5 in the Hossain/Makhnin document. Prove that the function

$$P_A : \Omega :\longrightarrow [0, \infty[\quad \text{defined as} \quad P_A(B) = P(AB) = P(A \cap B)$$

Satisfies all except one of the probability axioms.

In case you wondered: "$??$" in $(\Omega, ??, P)$ is a place holder for a σ-algebra $\mathscr{F} \subseteq \{A : A \subseteq \Omega\}$. This immaterial for this homework.