Math 330 Section 1 - Fall 2035 - Homework 03

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Last submission: Friday, September 12, 2025

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1; ch.2.1 - 2.6, 3.1 - 3.4 until Def.3.13

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 2.2

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Tuesday, September 2:

- **a.** Read carefully the remainder of MF ch.3.4.
- **b.** Read carefully MF ch.3.5. Draw plenty of pictures for Props.3.59 and 3.60 and Cor.3.4 (A = interval of real #s) Understand why inf(A) is also called the greatest lower bound of A and why it is a generalization of min(A). In ch.5 we will consider partial orderings. They are more general than the "linear" orderings of MF ch.4.
- **c.** Read carefully B/G ch.3 on logic. It is very brief, but all of it is extremely important.

Reading assignment 2 - due: Wednesday, September 3:

- **a.** Skim MF ch.4 on logic, just so you have an idea what's in there. Look a little bit more closely at ch.4.5.4 (Quantifiers and Negation). Note that I have marked all of ch.4 as optional, but you will be tested on B/G ch.3!

Reading assignment 3 - due Friday, September 5:

- **a.** Carefully read MF ch.5 through ch.5.2.4. (A First Look at Direct Images and Preimages of a Function.) You have already encountered much of the material on functions in ch.2.4.

Written assignments are on the next page.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:
Let $(R, \oplus, \odot)$ be an integral domain. Use anything up-to and including MF prop. 3.27 to prove MF prop.3.28: Let $x \in R$. If $x \odot x = x$ then $x = 0$ or $x = 1$.

Hint: Prove the following: If $x \odot x = x$ **and** $x \neq 0$ then $x = 1$. **Why is that enough**?

Written assignment 2: TOUGH! Get started EARLY!
Let $(R, \oplus, \odot, P)$ be an ordered integral domain. Use anything up-to and including MF prop. 3.34 to prove MF prop.3.35: The multiplicative unit 1 of R belongs to P.

Hint: This is an **indirect proof!** Part of it: Show that you cannot have $\ominus 1 \in P$. **Why** will this help you?

You are **strongly advised** to study the proof of Proposition 3.33 (newly added to MF version 2021-09-01) very thoroughly before working on this problem.