Math 330 Section 1 - Fall 2025 - Homework 04

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Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1 - 3, skim ch.4, ch.5 - 5.2.4

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 2.2, ch.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 8:

- **a.** Read carefully MF ch.5.2.5. Showing that some function is injective and/or surjective will be a major part of many proofs you will encounter later in this course.
- **b.** Read carefully MF ch.5.2.6 - 5.2.8 (the remainder of ch.5.2). Cartesian products as sets containing families as their elements requires getting used to!

Reading assignment 2 - due: Wednesday, September 10:

- **a.** Read carefully B/G ch.5 on sets and functions and B/G ch.9.1. You already have encountered the material in MF ch.2 and ch.5.1 - 5.2.
- **b.** The better students are challenged to take a look at the optional chapter 5.3.
- **c.** Read **extra carefully** MF ch.6.1. We briefly did induction already: We proved the triangle inequality. Proofs by induction will appear on all major exams!
- **d.** Read carefully MF ch.6.2 and 6.3. They are very brief.

Reading assignment 3 - due Friday, September 12:

- **a.** Read carefully B/G ch.2.3 on induction. Work through the proofs by induction given there.
- **b.** Carefully read MF ch.6.4 - 6.6.
- **c.** You are encouraged to look at the optional chapter 6.7 on Bernstein Polynomials. The proofs are very good examples of proofs by induction.

Written assignments are on the next page.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1: Use anything before Proposition 3.55 to prove the following part of Proposition 3.55:

Let $(R, \oplus, \odot, P)$ be an ordered integral domain. Let $A \subseteq R$. If A has a minimum then it also has an infimum, and $\min(A) = \inf(A)$.

Written assignment 2: Use the rules of working with quantifiers to negate the following statement (see B/G ch.3.3). No need at all to understand the meaning of this statement. [1]

$\forall\, \alpha > 0\; \exists\, \beta > 0$ such that $\forall\, x \in N_\beta(z)$ it is true that $f(x) \in N_\alpha(f(z))$.

[1] You will learn later in this course that this is the definition of continuity of a function $x \mapsto f(x)$ at a point z in the domain of f.