Math 330 Section 1 - Fall 2025 - Homework 08

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Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1 - 3, skim ch.4, ch.5 - 8

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 6.3, ch.9.1

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, October 6:

- **a.** Carefully read MF ch.9.1.
- **b.** Carefully read MF ch.9.2. Much of it is just a repeat of Ch.3.5.

Reading assignment 2 - due: Wednesday, October 8:

- **a.** Review Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition, you may have to search through the table of contents and/or consult the index.
- **b.** Extra carefully read MF ch.9.3. Convergence and continuity will be address in much more general settings in the chapters on metric and topological spaces and it is important that you are at ease with this material, including proofs!

Reading assignment 3 - due Friday, October 10:

- **a.** Read B/G ch.8. You already have encountered the material in MF ch.3 and 9.1 - 9.2.
- **b.** Carefully read MF ch.9.4 and 9.5.

Additional reading assignment (optional):

- **a.** If you have not taken or are not currently taking linear algebra, this would be a good time to look at MF ch.11 through Example 11.11. That's nine pages of very easy material.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignments are on the next page.

The written assignments are about proving MF thm.6.11 (Division Algorithm for Integers): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m \;=\; n \cdot q \;+\; r \quad \text{and } 0 \leq r < n.$$

Do not use induction for assignments 1 and 2. It would make your task more difficult!

Written assignment 1:

Prove uniqueness of the "decomposition" $m = qn + r$ such that $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by equating $qn + r \;=\; m \;=\; \tilde{q}n + \tilde{r}$ and see what can be said about $|r - \tilde{r}|$ if $q \neq \tilde{q}$. More hints further down!

Written assignment 2:

Much harder than #1: Prove the existence of q and r.

Hints for #2: Review the Extended Well–Ordering principle MF thm.6.10. Its use will give the easiest way to prove this assignment: Let

$$A \;:=\; A(m,n) \;:=\; \{r' \in [0, \infty[_{\mathbb{Z}} :\; \exists\, q' \in \mathbb{Z} \text{ such that } r' = m - q'n\}\,.$$

Show that $A \neq \emptyset$ by separately examining the cases

- $m \geqq 0$ (easy)
- $m < 0$ (probably the hardest part of the proof!)

Now you can apply the Extended Well–Ordering principle to the set A. How is $\min(A)$ related to your problem? (It is!) To better see what is going on, this may help:

- What is $m = nq + r$ for $n = 10$ and $m = 43,\; m = -43,\; m = -3$?
- What is $\min(A(m,n))$ in those three cases? Draw a picture!

Hint for both #1 and #2: MF Prop.3.60 and Cor.3.5 at the end of ch.3.5 will come in handy in connection with using or proving $0 \leqq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following.

If $a, b \in [0, n[_{\mathbb{Z}}$, then $|a - b| \;\leqq\; \max(a, b)$, i.e., $\;-n \;<\; a - b \;<\; n$.