Math 330 Section 1 - Fall 2025 - Homework 14

Published: Tuesday, November 11, 2025 Running total: 53 points
Last submission: Monday, December 1, 2025

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1 - 3, skim ch.4, ch.5 - 14.1 (skip ch.8.2 and 10.3)

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 7, ch.8-13.4

Other material (optional):
B/K lecture notes ch.1.1 and ch.1.2, except ch.1.2.4
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due Monday, November 17:

a. Read carefully MF ch.14.2 and 14.3. Understand the 3 equivalent ways to refer to compactness in metric spaces plus #4 in $\mathbb{R}^n$ Note that the "extract finite open subcovering" property needs extra careful study!

Reading assignment 2 - due: Wednesday, November 19:

a. Carefully read the remainder of MF ch.14.
b. Carefully read MF ch.15.1 and 15.3.

Reading assignment 3 - due Friday, November 21:

a. Review the end of MF ch.11.2.1, starting at Definition 11.10 (Linear dependence and independence).
b. Carefully read MF ch.15.2. and MF ch.15.4.
c. Carefully read MF ch.15.5.1. Very brief, but also abstract.

Written assignments on page 2

Written assignments:

Written assignment 1: Let X be a nonempty set. For each $j \in \mathbb{N}$ let $(x, y) \mapsto d_j(x, y)$ be a metric on X and let $a_j \in [0, \infty[$ such that $\sum_{j=1}^{\infty} a_j < \infty$ and at lease one a_j is not zero. Let

$$d(x, y) := \sum_{j=1}^{\infty} a_j \, d_j(x, y) \, ; \qquad (x, y \in X).$$

Prove that d defines a metric on X.

Written assignment 2: Prove Proposition 12.15:
Let $(X, \mathfrak{U})$ be a topological space If $A \subseteq B \subseteq X$, then $A^o \subseteq B^o$.

Written assignment 3: Let (X, d) be a metric space and $A \subseteq X$, $A \neq \emptyset$. Let

$$\gamma := \gamma(A) := \inf\{d(x, y) : x, y \in A \text{ and } x \neq y\}.$$

(a) Prove that if $\gamma > 0$ and $(x_n)_n$ is Cauchy in A, then $(x_n)_n$ is constant, eventually.
(b) Use **(a)** to prove that if $\gamma > 0$, then A is complete.
One point each for (a) and (b)!