Math 330 - Number Systems - Section 2 - Spring 2026

Selected Solutions for Spring 2026 Midterms

0.1 Midterm 1

Ver A #1; Ver B #5 (means problem 1 of Ver A which is problem 5 of Ver B):

#1a, #1b, #1c: See Lecture Notes **#1d**: $\boxed{\textbf{False}, \textbf{True}, \textbf{not}\, p \Rightarrow q,\ p \Rightarrow \textbf{not}\, q}$

Ver A #2; Ver B #6: See Lecture Notes, Ch.5.2.5

Ver A #3; Ver B #1: $\boxed{\forall\, r > 0\ \ \exists\, x \in X \text{ such that } \ \forall\, i \in I \quad f(r,x) > u_i}$

Ver A #4; Ver B #2: **Base cases:** $n = 0, 1$: $0 = 0 \cdot 3^0$; $3 = 1 \cdot 3^1$.

IA: have some n s.t. $x_j = j3^j$ for **all** $0 \le j \le n$. NTS $x_{n+1} = (n+1)3^{n+1}$.

Proof: LS = $x_{n+1} = 6x_n - 9x_{n-1} \overset{\textbf{IA}}{=} 6n3^n - 9(n-1)3^{n-1} = \ldots$ etc. = RS

Ver A #5; Ver B #7: **(a)** $h, h' \in H \Rightarrow h, h' \in H_i\ \forall\, i \Rightarrow h \diamond h' \in H_i\ \forall\, i \Rightarrow h \diamond h' \in \bigcap_i H_i$.

(b) $h \in H \Rightarrow h \in H_i\ \forall\, i \Rightarrow h^{-1}$ (= unique inverse in G) $\in H_i\ \forall\, i \Rightarrow h^{-1} \in \bigcap_i H_i$.

(c) e (= neutral elem. of G) $\in H_i\ \forall\, i \Rightarrow e \in \bigcap_i H_i$.

then $h^{-1} \in H$ **(c)** $e \in H$ ($e =$ neutral element of G)

Ver A #6; Ver B #3: $m := \max(A) \overset{\text{def.max}}{\Longrightarrow}$ **(a)**$m \in A$, **(b)**$m \in A_{upp\acute{b}}$.

From **(a)**: $m \le u\ \forall\, u \in A_{upp\acute{b}}$ From that + **(b)**: $m \overset{\text{def.min}}{=} \min(A_{upp\acute{b}})$. Thus, $m \overset{\text{def.sup}}{=} \sup(A)$.

Ver A #7; Ver B #4: Let $F(n) := n^3 + 2n$. Base case: True, since $F(1) = 3 = 1 \cdot 3$.

IA: Have some n for which $\exists j \in \mathbb{Z}$ s.t. $F(n) = 3j$. NTS: $\exists k \in \mathbb{Z}$ s.t. $F(n+1) = 3k$.

Proof: By simple arithmetic, $F(n+1) = \cdots = F(n) + 3(n^2 + n + 1) \overset{\textbf{(IA)}}{=} 3j + 3(n^2 + n + 1)$. Now, define $k := j + n^2 + n + 1)$. Done.