

Math 330 Section 1 - Spring 2026 - Homework 03

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Running total: 16 points

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:

ch.1; ch.2.1 - 2.6, 3.1 - 3.4 until Def.3.13

B/G (Beck/Geoghegan) Textbook:

ch.2.1 - 2.2

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Monday, February 2:

- a. Read carefully the remainder of MF ch.3.4.
- b. Read carefully MF ch.3.5. Draw plenty of pictures for Props.3.59 and 3.60 and Cor.3.4 (A = interval of real #s) Understand why $\inf(A)$ is also called the greatest lower bound of A and why it is a generalization of $\min(A)$. In ch.5 we will consider partial orderings. They are more general than the "linear" orderings of MF ch.4.
- c. Read carefully B/G ch.3 on logic. It is very brief, but all of it is extremely important.

Reading assignment 2 - due: Wednesday, February 4:

- a. Skim MF ch.4 on logic, just so you have an idea what's in there. Look a little bit more closely at ch.4.5.4 (Quantifiers and Negation). Note that I have marked all of ch.4 as optional, but you will be tested on B/G ch.3!

Reading assignment 3 - due Friday, February 6:

- a. Carefully read MF ch.5 through ch.5.2.4. (A First Look at Direct Images and Preimages of a Function.) You have already encountered much of the material on functions in ch.2.4.

Written assignments can be found on the next page.

Written assignments:

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Written assignment 1:

Let (R, \oplus, \odot) be an integral domain. Use anything up-to and including MF prop. 3.27 to prove MF prop.3.28: Let $x \in R$. If $x \odot x = x$ then $x = 0$ or $x = 1$.

Hint: Transform $x \odot x = x$ into a statement of the form $A \odot B = 0$ and do something that you can always do in an integral domain, but not necessarily in a CRU!

Written assignment 2: TOUGH! Get started EARLY!

Let (R, \oplus, \odot, P) be an ordered integral domain. Use anything up-to and including MF prop. 3.35 to prove MF prop.3.36: The multiplicative unit 1 of R belongs to P .

Hint: This is an **indirect proof!** Part of it: Show that you cannot have $\ominus 1 \in P$. **Why** will this help you?

You are **strongly advised** to study the proof of Proposition 3.33 very thoroughly before working on this problem.