# Math 330 Section 1 - Spring 2026 - Homework 08

Published: Tuesday, Feb 24, 2026 Running total: $30 + \_\_$ points
Last submission: Friday, March 13, 2026

**Status - Reading Assignments:**

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1 – ch.3, skim ch.4, ch.5.1 – ch.7

B/G (Beck/Geoghegan) Textbook:
ch.1 – 5, ch.9

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

**New reading assignments:**

**Reading assignment 1 - due Monday, March 2:**

- **a.** Carefully read MF ch.8.1. **You have been warned:** I love to ask the students in the major exams to prove (parts of) De Morgan!
- **b.** Those of you with an academic bend are encouraged to study the optional Chapter 8.2: $(2^\Omega, \triangle, \cap)$ as a CRU (Remark 8.1)**(5)**) (But $2^\Omega$ is not an integral domain: If $A, B$ are disjoint and nonempty, then $A \cap B = \emptyset$. Thus, $A$ and $B$ are zero divisors.)
- **c.** Carefully read MF ch.8.3. Be sure to understand formula (8.10): $Y^X = \{f : f \text{ is a function with domain } X \text{ and codomain } Y\}$.

**Reading assignment 3 - due Friday, March 6:**

- **a.** Carefully read MF ch.8.4 through Proposition 8.10. Skim or skip the remainder.
- **b.** Read MF ch.8.5. Indicator functions are very important for probability theory..

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

**Written assignments are on the next page.**

The written assignments are about proving MF thm.6.11 (Division Algorithm for Integers): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers $q$ ("quotient") and $r$ ("remainder") such that

$$m \;=\; n \cdot q \;+\; r \quad \text{and } 0 \leq r < n.$$

**a.** Do not use induction for assignments 1 and 2. It would make your task more difficult!

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**Written assignment 1:**

Prove uniqueness of the "decomposition" $m = qn + r$ such that $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by equating $qn + r \;=\; m \;=\; \tilde{q}n + \tilde{r}$ and see what can be said about $|r - \tilde{r}|$ if $q \neq \tilde{q}$. More hints further down!

**Written assignment 2:**

Much harder than #1: Prove the existence of $q$ and $r$.

**Hints for #2**: Review the Extended Well–Ordering principle MF thm.6.10. Its use will give the easiest way to prove this assignment: Let

$$A \;:=\; A(m,n) \;:=\; \{r' \in [0, \infty[_{\mathbb{Z}} :\; \exists\, q' \in \mathbb{Z} \text{ such that } r' = m - q'n\}\,.$$

Show that $A \neq \emptyset$ by separately examining the cases

- $m \geqq 0$ (easy)
- $m < 0$ (probably the hardest part of the proof!)

Now you can apply the Extended Well–Ordering principle to the set $A$. How is $\min(A)$ related to your problem? (It is!) To better see what is going on, this may help:

- What is $m = nq + r$ for $n = 10$ and $m = 43,\; m = -43,\; m = -3$?
- What is $\min(A(m,n))$ in those three cases? Draw a picture!

**Hint for both #1 and #2**: MF Prop.3.60 and Cor.3.5 at the end of ch.3.5 will come in handy in connection with using or proving $0 \leqq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following.

If $a, b \in [0, n[_{\mathbb{Z}}$, then $|a - b| \;\leqq\; \max(a,b)$, i.e., $\;-n \;<\; a - b \;<\; n$.