Math 330 Section 1 - Spring 2026 - Homework 10

Published: Tuesday, March 10, 2026 Running total: $34 +$ __ points
Last submission: Friday, March 27, 2026

Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1 – ch.3, skim ch.4, ch.5.1 – ch.9.5, ch.11 through Example 11.11

B/G (Beck/Geoghegan) Textbook:
ch.1 – 5, ch.8 – 9

Other material (optional):
B/K lecture notes ch.1.1 and ch.1.2, except ch.1.2.4
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due Monday, March 16:

- **a.** Carefully read B/G ch.6 and ch.7.1. This corresponds to material in MF ch.6.
- **b.** Carefully read B/G ch.10. This corresponds to material in MF ch.9.1 – 9.3.

Reading assignment 2 - due Wednesday, March 18:

- **a.** Carefully read MF ch.9.6 and 9.7.
- **b.** Carefully read B/G ch.11. This corresponds to material in MF ch.9.4.

Reading assignment 3 - due Friday, March 20:

- **a.** Carefully read MF ch.9.8 until before Proposition 9.44. and skim the optional remainder.
- **c.** Skip the optional MF ch.9.9 (Sequences of Sets and Indicator functions and their liminf and limsup). The stronger students and those who consider getting a master's degree in probability/statistics/actuarial science are encouraged to study the material, in particular the last remark.

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignments: See the next page!

Written assignments:

Written assignment 1: Prove Proposition 7.13: Every infinite set X contains a proper subset A that is countably infinite. **Hint:** • Define $X_0 := X \setminus \{a_0\}$ and then recursively, $X_n := X_{n-1} \setminus \{a_n\}$.

• Use an induction argument to prove that each set X_n is not empty (Thm.7.2 might come in handy). What does that imply for the number of elements of the set $\{a_0, a_1, \dots\}$? • How do you use those a_j to define the set A you want? • What is the bijection $A \xrightarrow{\sim}$?? which proves that A is indeed countably infinite?

Written assignment 2: Prove the following part of De Morgan's Law:

Let there be a universal set Ω which contains all elements of an indexed family of sets $(A_\alpha)_{\alpha \in I}$. Then

$$\Big(\bigcap_\alpha A_\alpha\Big)^{\complement} \subseteq \bigcup_\alpha A_\alpha^{\complement}.$$

Written assignment 3: Prove formula (8.33):

If X, Y, Z be arbitrary, nonempty sets and $f : X \to Y$, $g : Y \to Z$, and $U \subseteq X$, then

$$(g \circ f)(U) \subseteq g\big(f(U)\big) \text{ for all } U \subseteq X.$$