Math 330 Section 1 - Spring 2026 - Homework 11

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Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1 – ch.3, skim ch.4, ch.5.1 – ch.9.8, ch.11 through Example 11.11; skip ch.9.9

B/G (Beck/Geoghegan) Textbook:
ch.1 – 7.1, ch.8 – 11

Other material (optional):
B/K lecture notes ch.1.1 and ch.1.2, except ch.1.2.4
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due Monday, March 23:

- **a.** Carefully read MF ch.10.1 – 10.2. Unless you are a masochist, stay away from ch.10.3.
- **b.** Previously assigned: If you neither have taken nor are currently taking a linear algebra course, read carefully MF ch.11.1 and ch.11.2.1 through Example 11.11 (Vector spaces of real–valued functions).
  Otherwise, focus on the examples in MF ch.11.2.1. In particular, study the function space examples, e.g., Example 11.11 (Vector spaces of real–valued functions).
- **c.** Read the remainder of MF ch.11.2.1.

Reading assignment 2 - due Wednesday, March 25:

- **a.** Read VERY CAREFULLY MF ch.11.2.2. Skip nothing! Be sure to understand for $p = 2$ why $\|f\|_{L^p} = \left(\int_a^b |f(x)|^p dx \right)^{1/p}$ is a measure for the size of f. This will be easier if you draw a picture for $p = 1$!

Reading assignment 3 - due Friday, March 27:

- **a.** Read carefully B/G ch.12 and ch.13.1 – 13.4. You know the material from MF ch.7, 9, 10.
- **b.** Skip the optional MF ch.11.2.3 (The Inequalities of Young, Hoelder, and Minkowski). The stronger students and those interested in engineering or physics are encouraged to at least skim the contents.
- **c.** Read carefully MF ch.12.1 and ch.12.2. It is crucial that you become familiar with the examples given there. You will have massive problems with metric and topological spaces if you did not understand MF ch.9.3.

Be sure to read pages 2 and 3!

Supplementary instructions for reading MF ch.12:

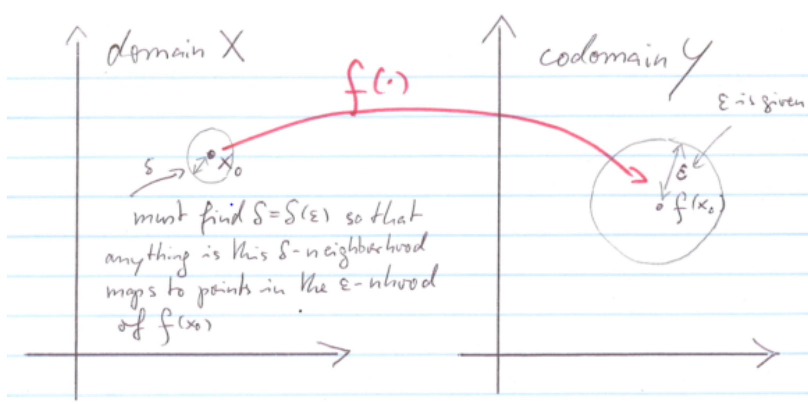
When you read or reread any topics in those chapters then the following is good advice:

a. MF ch.12.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for

- open sets and neighborhoods (ch.12.3)
- convergence, expressed with nhoods (the end of def.12.10 in ch.12.4)
- metric and topological subspaces (ch.12.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \biguplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^\complement$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^\complement$ and A_1. What is $N_\varepsilon^A(x_j)$?
- Contact points, closed sets and closures (ch.12.8): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^\complement$? Use those pictures to visualize the definitions in this chapter and thm 12.7 and thm.12.8.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.13.2 (Continuity): Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 0.1: ε-δ continuity



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Written assignments on page 3

Written assignments:

Written assignment 1: Prove formula (9.14) of prop.9.11: Let X be a nonempty set and $\varphi, \psi : X \to \mathbb{R}$. Let $\emptyset \neq A \subseteq X$. Then

$$\inf\{\varphi(x) + \psi(x) : x \in A\} \ \geqq\ \inf\{\varphi(y) : y \in A\} \ +\ \inf\{\psi(z) : z \in A\}.$$

Do the proof by modifying the proof of formula (9.13). Follow that proof as closely as possible!
You are **NOT ALLOWED** to apply formula (9.13) to $-\varphi$ and $-\psi$.

Written assignment 2: Prove MF prop.9.19**(b)**: If y_n is a sequence of real numbers that is non-increasing, i.e., $y_n \ \geqq\ y_{n+1}$ for all n, and bounded below, then $\lim\limits_{n\to\infty} y_n$ exists and coincides with $\inf\{y_n : n \in \mathbb{N}\}$. • Do the proof by modifying the proof of prop.9.19**(a)**.
You are **NOT ALLOWED** to apply prop.9.19**(a)** to the sequence $x_n := -y_n$!