Math 447 - Probability - Section 1 - Spring 2025

Selected Solutions for Spring 2025 Midterms

0.1 Midterm 1

Ver A #1; Ver B #3 (means problem 1 of Ver A which is problem 3 of Ver B):

1(a): $\boxed{\sigma\text{–algebra, probability measure}}$ **2(b, c, d)**: See lecture notes.

Ver A #2; Ver B #4: $\boxed{f \geq 0}$ and $\boxed{\iint_{\mathbb{R}^2} f(x,y)dxdy = 1}$

Ver A #3; Ver B #1:

3(a): $P\{\dots\} = P\{4\} + P\{5\} + P\{6\} = \boxed{\frac{7}{64}}$

3(b): $P_Y(\dots) = P\{1,2,3\} = \boxed{\frac{7}{8}}$

Ver A #4; Ver B #2: It was not explicitly stated, but the cards were meant to be non–distinct. If you assumed they were distinct (much harder!), you got quite generous credit, even if the solution was not even close to being correct:

4(a): $\binom{n+r-1}{r-1} = \binom{12}{3} = \boxed{220}$ **4(b)**: $\binom{n-1}{r-1} = \binom{8}{3} = \boxed{56}$

Ver A #5; Ver B #7: $1 = c \cdot \int_0^3 y_2^2 \left[\int_0^{y_2} y_1\, dy_1\right] dy_2 = \frac{3^5}{10} \Rightarrow \boxed{c = \frac{10}{243}}$.

Ver A #6; Ver B #8:

Solution for A:

	$\mathbf{Y_2}$		
$\mathbf{Y_1}$	0	1	2
0	1/9	2/9	1/9
1	2/9	2/9	0
2	1/9	0	0

Solution for B:

- $\vec{Y}^{-1}(\dots) = \boxed{\{(A,C),(C,A),(B,C),(C,B),(C,C),\}}$

Solution for C:

- $P_{\vec{X}}(\dots) = P_{\vec{X}}\{(A,C),(C,A),(B,C),(C,B),(C,C),\}$ $= \boxed{5/9}$

Ver A #7; Ver B #5:

7(a): $P(A) = \boxed{1/2}$ $P(A \mid A) = \boxed{1}$ $P(A \mid B) = \boxed{1}$ $P(A \mid C) = \boxed{1/2}$ $P(A \mid D) = \boxed{0}$

7(a): A and A **False** A and B **False** A and C **True** A and D **False**

Ver A #8; Ver B #6:

8(a): $A :=$ { $10, 20$ in at least one } $\Rightarrow$ $A^{\complement} =$ { neither of $10, 20$ happens in each one }.

Use independence: $P(A^{\complement}) = (22/24)^6$. Thus, $P(A) = \boxed{1 - \left(\frac{11}{12}\right)^6}$

Ver A #8; Ver B #6:

8(b): Since duplicates can happen, $|\Omega| = 24^6$. Since order matters, there are P_6^{24} distinct selections of size 6. Thus, $P\{\dots\} = \boxed{\frac{P_6^{24}}{24^6}}$