# Math 447 - Probability - Section 1 - Spring 2025

# Selected Solutions for Spring 2025 Quizzes

**Quiz 01:**

**#1 (a)**: Obvious choice for $\Omega$: $\Omega = \{LL,\ LM,\ LU,\ ML,\ MM,\ MU,\ UL,\ UM,\ UU\ \}$.

Then, $P\{\omega\} = \boxed{1/9,\ \forall \omega \in \Omega}$. **#1 (b)**: $P(A_1) = \boxed{5/9}$ **#1 (c)**: $P(A_2) = \boxed{5/9}$

**#3**: See Definition 2.5

**#4**: $\boxed{\{2k\pi : k \in \mathbb{Z}\} \cup \{\frac{\pi}{2} + 2k\pi : k \in \mathbb{Z}\}}$ (things are $2\pi$–periodic!)

**Quiz 02:**

**#1**: $\boxed{\{\ (\text{Yes}, 2),\ (\text{No}, 2),\ (\text{Yes}, 4),\ (\text{No}, 4),\ (\text{Yes}, 6),\ (\text{No}, 6),\ \}}$

**#2**: With De Morgan: The probability is $1\ -\ P(A \cap B)\ =\ 1 - (0.8 - 0.5)\ =\ \boxed{0.7}$

**#3**: $D$ is one half of the quad $[0,2] \times [0,2] \times [0,3]$ when one slices it diagonally with the plane that contains the points $(0,0,0)$ and $(0,2,2)$. Thus, $\lambda^3(D) = 6$. Thus, $\int g(\vec{x}) \cdot \mathbf{1}_D(\vec{x}) d\vec{x}\ =\ \boxed{30}$

**#4**: $f(x) = \boxed{0,\ \forall x}$ $\int\limits_{-\infty}^{\infty} f(x)dx = \boxed{0}$ $\int\limits_{-\infty}^{\infty} f_n(x)dx = \boxed{\infty,\ \forall n}$ $\lim\limits_{n\to\infty} \int\limits_{-\infty}^{\infty} f_n(x)dx = \boxed{\infty}$

**Quiz 03:**

**#1**: See lecture notes. **#2(a)**: **False** since, e.g., $A^{\complement} \notin \mathfrak{F}$ **#2(b)**: **True**

**#3(a)**: $(1/2)2\pi = \boxed{\pi}$ **#3(b)**: $5 \cdot 2 \cdot 2\ -\ 3 \cdot 1 \cdot 5\ =\ \boxed{5}$

**Quiz 04:**

**#1(a)**: $\boxed{[0,1]}$ **#1(b)**: $\boxed{\{\ \emptyset,\ [0,1],\ ]1,2],\ [0,2]\ \}}$

**#2(a)**: $\boxed{\{(0,0),\ (1,0)\}}$ **#2(b)**: $\boxed{\{(2,2)\}}$

**#3(a)**: $\boxed{S}$ **#3(b)**: $\boxed{\text{distribution}}$ **#3(c)**: $P(Y^{-1}(U))\ =\ \boxed{P\{Y \in U\}}$

**#4(a)**: $\boxed{0.4}$ **#4(b)**: $\boxed{0.5}$ **#4(c)**: **True** **#4(d)**: **True**

**Quiz 05:**

**#1**: $\binom{29}{9, 12, 8}\ =\ \boxed{\frac{29!}{9! \cdot 12! \cdot 8!}}$ **#2**: $\binom{8}{4}\ =\ \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2}\ =\ \boxed{70}$

**#3**: $\boxed{\frac{P_7^{10}}{10^7}}$ **The reason:** There are $10^7$ different vectors $(x_1, x_2, \ldots, x_7)$ such that each $x_j \in [1,10]_{\mathbb{Z}}$.

Of those, $P_7^{10}\ =\ 10 \cdot 9 \cdots 8$ have no duplicates.

**#4(a)**: $\boxed{\binom{88}{20}}$ **#4(b)**: $\dfrac{\binom{36}{12}\cdot\binom{88-36}{20-12}}{\binom{88}{20}} = \boxed{\dfrac{\binom{36}{12}\cdot\binom{52}{8}}{\binom{88}{20}}}$

**Quiz 06:**

**#1**: True: $\boxed{\text{random samples}}$. The other 3 are false **#2**: See lecture notes

**#3**: **(a)** $\boxed{\frac{2}{3}}$ **(b)** $\boxed{\frac{1}{3}}$ **#4**: P(no contact | nonfiction) $= \boxed{\frac{4}{7}}$ (tree diagram!)

**Quiz 07:**

**#1(a)**: $\boxed{\sum_y y \cdot p_Y(y)}$ **#1(b)**: $\boxed{\sum_y \frac{3y+5}{10y^2+1} \cdot p_Y(y)}$

**#2**: **(a)** $\boxed{12}$ $\sqrt{18\cdot(2/3)\cdot(1/3)} = \sqrt{(36/9)} = \boxed{2}$

**b.** $\boxed{\frac{3}{2}}$ $1-(6/9)-(2/3)(1/3) = \boxed{\frac{1}{9}}$ **c.** $\boxed{9}$ $\boxed{9}$

**#3**: **(a)** $\boxed{7}$ **(b)** $\boxed{2}$ **(c)** $4^2+5^2 = \boxed{41}$ **(d)** $Var[V] = 4\cdot 4^2+5^2 = \boxed{89}$

**#4**: **(a)** $10.00 \mapsto \boxed{\frac{1}{4}}$ $5.00 \mapsto \boxed{\frac{1}{12}}$ $0.00 \mapsto \boxed{\frac{2}{3}}$ **(b)** $10(3/12)+5(1/12)+0(8/12)-3.00 = \boxed{\frac{-1}{12}}$