Math 447 - Spring 2025 - Homework 02

Published: Thursday, January 23, 2025

Status - Reading Assignments:

Here are the reading assignments to be completed before the first one of this HW.

WMS (Wackerly, et al. Textbook):
Nothing assigned yet

MF447 lecture notes:
ch.1 - 2.3

Other:
Nothing assigned yet

New reading assignments:

It is really important for the WMS reading assignments that you work through the examples!

Reading assignment 1 - due Monday, January 27:

- **a.** Extra carefully read MF ch.2.4. You may find it very difficult if you did not attend Math 330 Remember that I talked in Ch.1 about the assignment $A \mapsto P(A)$ of a probability measure as a function $2^{\Omega} \to [0, 1]$.
- **b.** Carefully read the remainder of MF ch.2. Note that ch.2.5 (Preimages) is new in MF ver 2025-01-23!

Reading assignment 2 - due: Wednesday, January 29:

- **a.** Carefully read MF ch.3 through Example 3.4 in Ch.3.2.1. You should understand how this material relates to Calc 2 and Calc 3. You (should) have learned the material in Calc2 and Calc3. Only the notation is quite different.

Reading assignment 3 - due Friday, January 31:

- **a.** Carefully read the remainder of MF ch.3. You should understand how this material relates to Calc 2 and Calc 3.
- **b.** Carefully read the few non–optional parts of MF ch.4.1. You will strongly benefit if you understand from the examples and remarks the content of the definitions and theorems. Only the strong students are encouraged to look at the proofs.

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

Written assignments are on the next page.

Written assignments - Not collected for grading:

Remember that **some of those assignments will be relevant for the quizzes and exams.**

- Written assignments were re–published on (Wed 8/28).

(a) Write rom memory the following definitions and compare them with the MF lecture notes:

- countable set
- probability space def. with σ–additivity
- outcome vs event
- min/max/inf/sup, $1_A(\omega)$
- Review Stewart multivariable calculus: double and triple integrals. Also for Calc 2 integrals: What are Riemann sums? What are step functions? How is that used to define $\int f(\vec{x})d\vec{x}$ for $\vec{x} \in \mathbb{R}^d$, $(d = 1, 2, 3)$?

(b) Two dice are rolled at random; What is a probability space for events that correspond to the (potential) outcomes (e.g, $(3, 1)$ means Y_1 = first roll = 3, Y_2 = 2nd roll = 1) of those rolls? What is $P\{8 \leq Y_1 + Y_2 \leq 10\}$? Use brute force: arrange all outcomes into a square grid and check off those with a sum between 8 and 10.

(c) Answer the following questions

(c1) **Q:** For what kind of series does each rearrangement give the same value:

(c2) **Q:** Are there any kinds of unions where rearranging the sets gives different results? What about intersections?

(c3) If $A_1 \subseteq A_2 \subseteq \cdots$, what is $\min_j 1_{A_j}, \inf_j 1_{A_j}, \max_j 1_{A_j}, \sup_j 1_{A_j}$?
If $A_1 \supseteq A_2 \supseteq \cdots$, what is $\min_j 1_{A_j}, \inf_j 1_{A_j}, \max_j 1_{A_j}, \sup_j 1_{A_j}$?

(d) One of the following assignments defined on the atomar events n of the sample space $\mathbb{N}$ can be extended to a probability measure on $\mathfrak{F} := 2^{\mathbb{N}}$. Which one? What is wrong with the other two?

- $\{n\} \mapsto P_1\{n\} := (1/2)^{n-1}(1/4)$
- $\{n\} \mapsto P_2\{n\} := (1/2)^{n-1}(1/2)$
- $\{n\} \mapsto P_3\{n\} := (1/2)^{n-1}(3/4)$

Selected answers:

Answers for **(c)**:

(c1) See MF ch.3.1

(c2) Rearrangements can never affect nions or intersections. (MF ch.2)

(c3) If $A_1 \subseteq A_2 \subseteq \cdots$, $\min_j 1_{A_j} = \inf_j 1_{A_j} = A_1$; $\max_j 1_{A_j}$ DNE,(in general); $\sup_j 1_{A_j} = 1_{\bigcup_j A_j}$.
If $A_1 \supseteq A_2 \supseteq \cdots$, $\max_j 1_{A_j} = \sup_j 1_{A_j} = A_1$; $\min_j 1_{A_j}$ DNE,(in general); $\inf_j 1_{A_j} = 1_{\bigcap_j A_j}$.

(d) Since $\mathbb{N} = \{1\} \uplus \{2\} \uplus \{3\} \uplus \cdots$, We must have $P_1(\mathbb{N}) = P_2(\mathbb{N}) = P_3(\mathbb{N}) = 1$. Let $q := 1/2$:

$$\sum_{j=0}^{\infty} q^j = \frac{1}{1-1/2} = 2\,.$$

Thus,

$$\sum_{j=1}^{\infty} P_1\{j\}q^j = \frac{1}{4}\sum_{j=1}^{\infty} q^{j-1} = \frac{1}{4}\sum_{j=0}^{\infty} q^j = \frac{2}{4} \neq 1\,.$$

Likewise,

$$\sum_{j=1}^{\infty} P_2\{j\}q^j = \frac{1}{2}\sum_{j=1}^{\infty} q^{j-1} = \frac{2}{2} = 1\,,$$
$$\sum_{j=1}^{\infty} P_3\{j\}q^j = \frac{3}{4}\sum_{j=1}^{\infty} q^{j-1} = \frac{3\cdot 2}{4} \neq 1\,.$$

Only P_2 can be extended to a probability measure on $2^{\mathbb{N}}$.