Math 447 - Fall 2025 - Homework 02

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Status - Reading Assignments:

Here are the reading assignments to be completed before the first one of this HW.

WMS (Wackerly, et al. Textbook):

Nothing assigned yet

MF447 lecture notes:

ch.1 - 2.3

Other:

Nothing assigned yet

New reading assignments:

It is really important for the WMS reading assignments that you work through the examples!

Reading assignment 1 - due: Wednesday, August 27:

- **a.** Carefully read MF ch.2.4 and 2.5. Most of this has already been covered in lecture.
- **b.** Carefully read the remainder of MF ch.2. The material is easy to grasp. Note that I will work quite extensively with indicator functions when we do integration and expectations of random variables!

Reading assignment 2 - due Friday, August 29:

- **a.** Carefully read MF ch.3.1. The non optional material should be known from calculus.
- **b.** Carefully read MF ch.3.2. The notation may not be familiar, but you also should know that material from calculus.

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

(a) Write rom memory the following definitions and compare them with the MF lecture notes:

- countable set
- probability space def. with σ–additivity
- outcome vs event
- min/max/inf/sup, $1_A(\omega)$
- Review Stewart multivariable calculus: double and triple integrals. Also for Calc 2 integrals: What are Riemann sums? What are step functions? How is that used to define $\int f(\vec{x})d\vec{x}$ for $\vec{x} \in \mathbb{R}^d$, $(d = 1, 2, 3)$?

(b) Two dice are rolled at random; What is a probability space for events that correspond to the (potential) outcomes (e.g, $(3, 1)$ means Y_1 = first roll = 3, Y_2 = 2nd roll = 1) of those rolls? What is $P\{8 \leq Y_1 + Y_2 \leq 10\}$? Use brute force: arrange all outcomes into a square grid and check off those

with a sum between 8 and 10.

(c) Answer the following questions

(c1) **Q:** For what kind of series does each rearrangement give the same value:

(c2) **Q:** Are there any kinds of unions where rearranging the sets gives different results? What about intersections?

(c3) If $A_1 \subseteq A_2 \subseteq \cdots$, what is $\min_j 1_{A_j}, \inf_j 1_{A_j}, \max_j 1_{A_j}, \sup_j 1_{A_j}$?
If $A_1 \supseteq A_2 \supseteq \cdots$, what is $\min_j 1_{A_j}, \inf_j 1_{A_j}, \max_j 1_{A_j}, \sup_j 1_{A_j}$?

(d) One of the following assignments defined on the atomar events n of the sample space $\mathbb{N}$ can be extended to a probability measure on $\mathfrak{F} := 2^{\mathbb{N}}$. Which one? What is wrong with the other two?

- $\{n\} \mapsto P_1\{n\} := (1/2)^{n-1}(1/4)$
- $\{n\} \mapsto P_2\{n\} := (1/2)^{n-1}(1/2)$
- $\{n\} \mapsto P_3\{n\} := (1/2)^{n-1}(3/4)$

Selected answers:

Answers for **(c)**:

(c1) See MF ch.3.1

(c2) Rearrangements can never affect nions or intersections. (MF ch.2)

(c3) If $A_1 \subseteq A_2 \subseteq \cdots$, $\min_j 1_{A_j} = \inf_j 1_{A_j} = A_1$; $\max_j 1_{A_j}$ DNE,(in general); $\sup_j 1_{A_j} = 1_{\bigcup_j A_j}$.
If $A_1 \supseteq A_2 \supseteq \cdots$, $\max_j 1_{A_j} = \sup_j 1_{A_j} = A_1$; $\min_j 1_{A_j}$ DNE,(in general); $\inf_j 1_{A_j} = 1_{\bigcap_j A_j}$.

(d) Since $\mathbb{N} = \{1\} \biguplus \{2\} \biguplus \{3\} \biguplus \cdots$, We must have $P_1(\mathbb{N}) = P_2(\mathbb{N}) = P_3(\mathbb{N}) = 1$. Let $q := 1/2$:

$$\sum_{j=0}^{\infty} q^j \;=\; \frac{1}{1-1/2} \;=\; 2\,.$$

Thus,

$$\sum_{j=1}^{\infty} P_1\{j\} q^j \;=\; \frac{1}{4} \sum_{j=1}^{\infty} q^{j-1} \;=\; \frac{1}{4} \sum_{j=0}^{\infty} q^j \;=\; \frac{2}{4} \neq 1\,.$$

Likewise,

$$\sum_{j=1}^{\infty} P_2\{j\} q^j \;=\; \frac{1}{2} \sum_{j=1}^{\infty} q^{j-1} \;=\; \frac{2}{2} = 1\,,$$

$$\sum_{j=1}^{\infty} P_3\{j\} q^j \;=\; \frac{3}{4} \sum_{j=1}^{\infty} q^{j-1} \;=\; \frac{3 \cdot 2}{4} \neq 1\,.$$

Only P_2 can be extended to a probability measure on $2^{\mathbb{N}}$.