Formula Collection for Math 447 Midterm 1, Spring 2026 – Not all items are relevant!

Advice: Study this sheet before the exam, so you know where to look if you need something!

Abbreviations: • rv = r.v. = random variable; r.e. = random element • prob = probability • distr = distribution • cond = conditional • cond$\mathbb{P}$ = cond probab • func = function • conv = convergence • cont = continuous/continuity • abs = absolute • w.r.t = with respect to • def'd = defined • discr = discrete • indep = independent, independence • $\int$-ble = integrable

(1) **(a)** • power set $2^\Omega = \{$ all subsets of $\Omega\}$ • $\forall x \dots$: For all $x \dots$ ⊡ $\exists x$ s.t. $\dots$ There is an x such that $\dots$ ⊡ $\exists! x$ s.t. $\dots$ There is unique x s.t. $\dots$ ⊡ $p \Rightarrow q$: If p is true then q is true ⊡ $p \Leftrightarrow q$ p: iff q, i.e., p true if and only if q true
• Intervals: $]a,b[= \{x \in \mathbb{R} : a < x < b\}$, $]a,b]_\mathbb{Z} = \{x \in \mathbb{Z} : a < x \le b\}$, $[a,b]_\mathbb{Q} = \{x \in \mathbb{Q} : a \le x \le b\}$, etc.
• countable set A: can be sequenced: ⊡ $A = \{a_1, a_2, \dots, a_n\}$ (finite set) ⊡ $A = \{_1, a_2, \dots\}$ ("countably infinite" set) ⊡ $\mathbb{Z}$ and $\mathbb{Q}$ are countable, but $\mathbb{R}$ is uncountable • family $(x_i)_{i\in I}$: index set I may be uncountable • $\bigcup_{i\in J} A_i = \{x : \exists\, i_0 \in J \text{ s.t. } x \in A_{i_0}\}$ • $\bigcap_{i\in J} A_i = \{x : \forall\, i \in J\, x \in A_i\}$. • Can use $A \biguplus B$ for $A \cup B$ if disjoint sets • **Distributivity:** ⊡ $\bigcup_j (B \cap A_j) = B \cap \bigcup_j A_j$ ⊡ $\bigcap_{j\in I} (B \cup A_j) = B \cup \bigcap_j A_j$ • **De Morgan:** ⊡ $\left(\bigcup_k A_k\right)^\complement = \bigcap_k A_k^\complement$ ⊡ $\left(\bigcap_k A_k\right)^\complement = \bigcup_k A_k^\complement$
• Cartesian products: $\left| X_1 \times \cdots \times X_n \right| = |X_1| \cdots |X_n|$ (combinatorics counting rule) • Preimages of $f : X \to Y$ satisfy: ⊡ Arbitrary index set J and $B, B_j \subseteq Y$: ⊡ $f^{-1}(\bigcap_{j\in J} B_j) = \bigcap_{j\in J} f^{-1}(B_j)$ ⊡ $f^{-1}(\bigcup_{j\in J} B_j) = \bigcup_{j\in J} f^{-1}(B_j)$
⊡ $f^{-1}(B^\complement) = \left(f^{-1}(B)\right)^\complement$ ⊡ $B_1 \cap B_2 = \emptyset \Rightarrow f^{-1}(B_1) \cap f^{-1}(B_2) = \emptyset$ • $A \subseteq \Omega \Rightarrow \mathbf{1}_A(\omega) = 1$ if $\omega \in A$ and 0 else
• partition B_j $(j \in \mathbb{N})$ of Ω, $A \in \Omega \Rightarrow A = \biguplus_j (A \cap B_j) \Rightarrow P(A) = \dots;$

(b) Sums and Riemann integrals (Riem-$\int$) = $\int_A f(\vec{y})d\vec{y}$ and Lebesgue integrals (Leb-$\int$) = $\int_A f d\lambda^d$:
• $x_n \ge 0$ or $\sum_n x_n$ abs conv $\Rightarrow \sum_n x_n$ satisfies WHAT? • Leb-$\int$: positive, monotone, linear; mon. + domin. conv.
• step function h: $\int h(\vec{y})d\vec{y} =?$ • simple function g: $\int g d\lambda^d =?$ • If both $\int_A f(\vec{y})d\vec{y}, \int_A f d\lambda^d$ exist, they are equal
• Use Fubini for both $\int_A f(\vec{y})d\vec{y}$ and $\int f d\lambda^d$ to compute multidim $\int$. • $\mathbf{1}_A$ Riem-$\int$-ble $\Rightarrow \lambda^d(A)$ def'd how?
• $\mathbb{1}_A$ **=?** • $\mathfrak{B}^d = \sigma\{$ d–dim rectangles $\}$ • $\left[f \ge 0 \text{ or } \int |f| d\lambda^d < \infty\right] \Rightarrow \left[A \mapsto \int_A f\, d\lambda^d \text{ is } \sigma\text{–additive}\right]$
• For what func φ is $A \mapsto \sum_{\omega\in A} \varphi(\omega)$ a prob meas? • For what func ψ is $A \mapsto \int_A \psi(\vec{y})d\vec{y}\ \left(= \int_A \psi d\lambda^d\right)$ a prob meas?

(c) • prob space = sample space $(\Omega, \mathfrak{F}, P)$ • $\mathfrak{F} \subseteq 2^\Omega$ def'd how? • distr P_X of r.e. $X : (\Omega, P) \to \Omega'$ def'd how?
• Conveniences: ⊡ $\{X = x\} = X^{-1}(\{x\})$; ⊡ $\{X \in B\} = X^{-1}(B)$; ⊡ if Y is r.v., i.e., $\Omega' \subseteq \mathbb{R}$: $\{a < Y \le b\} = Y^{-1}\left(]a,b]\right)$
• discr prob spaces and r.e.s and r.v.s def'd how?

• indep for $2, n$, arbitr. many events $(A_i)_i$ or r.e.s $(X_i)_i$ • $P(A \mid B), P(X \in A \mid B), P(A \mid X \in B), \dots$
• Addition rule; multiplication rule for cond$\mathbb{P}$s; complement rule

(d) Combinatorial Analysis • Think: Does order matter in your prob space or doesn't it?
• multiplication rule (= counting rule) for several factors • # of permutations P^n_r vs # of combinations $\binom{n}{r}$
⊡ $0! = 1, n! = 1 \cdot 2 \cdots n;\ (n \in \mathbb{N})$

• deck of 52 cards: ⊡ 4 suits (clubs, spades, hearts, diamonds) of 13 each: Ace, $2, 3, \dots, 10$, Jack, Queen, King ⊡ so: 4 2's, 4 3's, 4 Aces, 4 Jacks, $\dots$ • Roulette: ⊡ slots $0, 00, 1, 2, \dots, 36$ ⊡ 18 black, 18 red; numbers $1 - 36$ in 12 rows $\times$ 3 cols