

## Math 447 - Spring 2026 - Homework 03

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### Status - Reading Assignments:

Here are the reading assignments to be completed before the first one of this HW.

MF447 lecture notes:

ch. 1 - 4 (Problem 4.1)

WMS (Wackerly, et al. Textbook):

Nothing assigned yet

Other:

Nothing assigned yet

### New reading assignments:

It is really important for the WMS reading assignments that you work through the examples!

#### Reading assignment 1 - due Monday, February 2:

- a. Carefully read the remainder of MF ch.4. Of particular importance for understanding the reason why we work with the Lebesgue integral rather than the Riemann integral is Remark 4.12.

#### Reading assignment 2 - due Wednesday, February 4:

- a. Carefully read MF ch.5.1 through Corollary 5.1.

#### Reading assignment 3 - due Friday, February 6:

- a. Carefully read the remainder of MF ch.5.1.
- b. Carefully read MF ch.5.2. It's material is of immense importance for attacking real world problems (and those posed in the quizzes and exams of this course).

**General note on written assignments:** I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

(a1) Write from memory the following definitions and compare them with the MF lecture notes:

- countable set
- probability space def. with  $\sigma$ -additivity
- outcome vs event
- min/max/inf/sup, cartesian product,  $1_A(\omega)$
- Review Stewart multivariable calculus: double and triple integrals. Also for Calc 2 integrals: What are Riemann sums? What are step functions? How is that used to define  $\int f(\vec{x})d\vec{x}$  for  $\vec{x} \in \mathbb{R}^d$ , ( $d = 1, 2, 3$ )?

(a2) Work closed book through the examples given in Section 3.4 (Series and Integrals as Tools to Compute Probabilities). All integrals in there are given as Riemann integrals.

(a3) Write from memory the following definitions and compare them with the MF lecture notes:

- Step functions and simple functions
- Lebesgue integral for  $f \geq 0$ . Can you draw a picture that shows how such  $f$  is approximated from below by simple functions?
- Properties of the Riemann integral.
- Properties of the Lebesgue integral.
- Monotone and dominated convergence theorems. Write them from memory until you get the assumptions and conclusions right for both of them!
- Really important: Work through the examples given in Problem 4.1.
- Write Fubini's theorems from memory for both Riemann integral and Lebesgue integral.
- $\sigma$ -algebras

If you are short on time, push the remainder of (a3) to the next weekend:

- probability measures and probability spaces (ch.5 definition!) • equiprobability • Continuity property of probability measures (optional, but helpful)
- discrete probability spaces • Theorem 5.2 and Cor.5.1
- Read again Remark 5.9. In part (d'), understand the cases  $n = 2$  and  $n = 3$ .
- Review the (optional) Fact 5.1. You will not be quizzed on it, but you want to understand it.
- Additive law of probability and the rule of the Complement
- Work Remark 5.12 closed book!
- Conditional probability • Multiplicative Law of Probability • Independence of 2, 3,  $n$  events and of sequences of events

(b) Two dice are rolled at random; What is a probability space for events that correspond to the (potential) outcomes (e.g, (3, 1) means  $Y_1 =$  first roll = 3,  $Y_2 =$  2nd roll = 1) of those rolls? What is  $P\{8 \leq Y_1 + Y_2 \leq 10\}$ ? Use brute force: arrange all outcomes into a square grid and check off those with a sum between 8 and 10.

(c) Answer the following questions

- (c1) **Q:** For what kind of series does each rearrangement give the same value?
- (c2) **Q:** Are there any kinds of unions where rearranging the sets gives different results? What about intersections?
- (c3) If  $A_1 \subseteq A_2 \subseteq \dots$ , what is  $\min_j 1_{A_j}, \inf_j 1_{A_j}, \max_j 1_{A_j}, \sup_j 1_{A_j}$ ?  
If  $A_1 \supseteq A_2 \supseteq \dots$ , what is  $\min_j 1_{A_j}, \inf_j 1_{A_j}, \max_j 1_{A_j}, \sup_j 1_{A_j}$ ?

(d) One of the following assignments defined on the atomic events  $n$  of the sample space  $\mathbb{N}$  can be extended to a probability measure on  $\mathfrak{F} := 2^{\mathbb{N}}$ . Which one? What is wrong with the other two?

- $\{n\} \mapsto P_1\{n\} := (1/2)^{n-1}(1/4)$
- $\{n\} \mapsto P_2\{n\} := (1/2)^{n-1}(1/2)$
- $\{n\} \mapsto P_3\{n\} := (1/2)^{n-1}(3/4)$

**Selected answers:**

Answers for (c):

(c1) See MF ch.3.1

(c2) Rearrangements can never affect unions or intersections. (MF ch.2)

(c3) If  $A_1 \subseteq A_2 \subseteq \dots$ ,  $\min_j 1_{A_j} = \inf_j 1_{A_j} = 1_{A_1}$ ;  $\max_j 1_{A_j}$  DNE,(in general);  $\sup_j 1_{A_j} = 1_{\cup_j A_j}$ .  
If  $A_1 \supseteq A_2 \supseteq \dots$ ,  $\max_j 1_{A_j} = \sup_j 1_{A_j} = 1_{A_1}$ ;  $\min_j 1_{A_j}$  DNE,(in general);  $\inf_j 1_{A_j} = 1_{\cap_j A_j}$ .

(d) Since  $\mathbb{N} = \{1\} \uplus \{2\} \uplus \{3\} \uplus \dots$ , We must have  $P_1(\mathbb{N}) = P_2(\mathbb{N}) = P_3(\mathbb{N}) = 1$ . Let  $q := 1/2$ :

$$\sum_{j=0}^{\infty} q^j = \frac{1}{1 - 1/2} = 2.$$

Thus,

$$\sum_{j=1}^{\infty} P_1\{j\}q^j = \frac{1}{4} \sum_{j=1}^{\infty} q^{j-1} = \frac{1}{4} \sum_{j=0}^{\infty} q^j = \frac{2}{4} \neq 1.$$

Likewise,

$$\sum_{j=1}^{\infty} P_2\{j\}q^j = \frac{1}{2} \sum_{j=1}^{\infty} q^{j-1} = \frac{2}{2} = 1,$$
$$\sum_{j=1}^{\infty} P_3\{j\}q^j = \frac{3}{4} \sum_{j=1}^{\infty} q^{j-1} = \frac{3 \cdot 2}{4} \neq 1.$$

Only  $P_2$  can be extended to a probability measure on  $2^{\mathbb{N}}$ .