Math 454 - Spring 2025 - Homework 02

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Status - Reading Assignments:

Here are the reading assignments to be completed before the first one of this HW.

SCF2 (Shreve – Stoch. Calculus for Finance, II Textbook):
 Ch. 1.1 – 1.5

MF454 lecture notes:
 Ch.2 – 3

Other:
 Nothing assigned yet

New reading assignments:

In the following, MF and MF454 both refer to my course lecture notes. See the course materials page. SCF2 refers to the Shreve text (Stochastic Calculus for Finance II), WMS = Wackerly, et al: the standard Math 447 Textbook.

Do not skip the remarks and examples in the assigned readings!

Reading assignment 1 - due Monday, January 27:

- **a.** Carefully read MF ch.4.1.

Reading assignment 2 - due Wednesday, January 29:

- **a.** Carefully read MF ch.4.2.

Reading assignment 3 - due Friday, January 31:

- **a.** Carefully read MF ch.4.3.
- **b.** Carefully read MF ch.4.4 through Proposition 4.14.

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

Written assignment 1:

- **a.** Do a proof "by picture" of Proposition 2.1.: Draw the Venn diagrams and convince yourself that they match.
- **b.** Do the same for Proposition 2.4 **b–h**.

Written assignment 2:

- **a.** Prove closed book formula (2.13).**a**: $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$.
- **b.** Theorem 3.1 has the general formula for arbitrary families: Prove that $\left(\bigcup\limits_{\alpha} A_\alpha\right)^{\complement} = \bigcap\limits_{\alpha} A_\alpha^{\complement}$.

Written assignment 3:

Remark 2.8 states that $x = x^+ - x^-$, $|x| = x^+ - x^-$, $f = f^+ - f^-$, $|f| = f^+ - f^-$. Prove it.

Written assignment 4:

a. Write the function $f : [-1,2[\to \mathbb{R};$ as a family.

b. Write the family $([x,2x])_{x \in \mathbb{Q}}$ as a function. In particular, what are domain and codomain?

Written assignment 5:

a. Pick at least two parts each from Examples 3.4–3.7 and verify the answers.

b. Prove closed book formula (3.16): Assume that X, Y be nonempty, $f : X \to Y$, J is an arbitrary index set, $B \subseteq Y$, $B_j \subseteq Y$ for all j. Then $f^{-1}(\bigcap_{j \in J} B_j) = \bigcap_{j \in J} f^{-1}(B_j)$

For the answer, see ch.8 of my Math 330 lecture notes for either Fall 2020 or Fall 2022.

Written assignment 6:

a. Do closed book the proof of Proposition 4.3 If $\mathfrak{E}_1 \subseteq \mathfrak{E}_2$, then $\sigma(\mathfrak{E}_1) \subseteq \sigma(\mathfrak{E}_2)$.

b. Do closed book the proof of Proposition 4.4 If $\mathfrak{E}_1 \subseteq \sigma(\mathfrak{E}_2)$, and $\mathfrak{E}_1 2 \subseteq \sigma(\mathfrak{E}_1)$, then $\sigma(\mathfrak{E}_1) = \sigma(\mathfrak{E}_2)$.

Selected Solutions:

Solution to #4:

a. $\big(f(x)\big)_{x \in [-1,2[}$ (a family in $\mathbb{R}$)

b. $f : \mathbb{Q} \to 2^{\mathbb{R}};\ f(x) := [x, 2x]$