Math 454 - Spring 2025 - Homework 03

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Status - Reading Assignments:

Here are the reading assignments to be completed before the first one of this HW.

SCF2 (Shreve – Stoch. Calculus for Finance, II Textbook):

Ch. 1.1 – 1.5

MF454 lecture notes:

Ch.2 – 3, ch.4.1 – 4.4, Proposition 4.14

Other:

Nothing assigned yet

New reading assignments:

In the following, MF and MF454 both refer to my course lecture notes. See the course materials page. SCF2 refers to the Shreve text (Stochastic Calculus for Finance II), WMS = Wackerly, et al: the standard Math 447 Textbook

Reading assignment 1 - due Monday, February 3:

- **a.** Carefully continue reading MF ch.4 through ch.4.6. Ch.4.5 and ch.4.6 contain the key material on integration and thus, on expectations. You will have trouble understanding conditional expectations, in particular, with respect to sub–σ–algebras, if you do not properly understand that subject.
- **b.** Carefully read MF ch.4.7.

Reading assignment 2 - due: Wednesday, February 5:

- **a.** Carefully read MF ch.4.8 and skim ch.4.9, but be sure to understand how it relates to what you learned about solving integrals $\int f(\vec{x})d\vec{x}$ by iterating onedimensional integrals.
- **b.** Carefully read MF ch.4.10. All of it should be familiar from Math 447.

Reading assignment 3 - due Friday, February 7:

- **a.** Carefully read MF ch.5.1. The proofs are not important, but extra carefully look at the remarks that interpret σ–algebras as stores of information and how Doob's Lemma relates to that subject!
- **b.** Extra carefully read MF ch.5.2. You must understand conditioning w.r.t σ–algebras generated by countable partitions to understand ch.5.3! Proposition 5.3. E(EX | G) = EX p.128
- **b.** Carefully read MF ch.5.3 until before Proposition 5.3. Understand how similar the rules for conditional expectations are to those for integrals and thus, ordinary expectations.

Written assignments are on the next page.

Written assignments:

General note on written assignments: I will not collect those assignments for grading but doing them might be helpful for your quizzes and exams.

Written assignment 1:

Prove closed book Theorem 4.1: Let $f : (\Omega, \mathfrak{F})$ and $(\Omega', \mathfrak{F}')$ and $\mathfrak{E}' \subseteq \mathfrak{F}'$ such that $\sigma(\mathfrak{E}') = \mathfrak{F}'$. Then, $f^{-1}(A') \subseteq \mathfrak{F}$ for all$A' \in \mathfrak{E}' \Rightarrow f \in m(\mathfrak{F}, \mathfrak{F}')$.

Written assignment 2:

a. In the proof of Proposition 4.11, the assertion $f(\omega) < g(\omega) \Leftrightarrow$ there is (at least one) $q \in \mathbb{Q}$ such that $f(\omega) < q < g(\omega)$ is made. Prove it!
Hint: • For any real numbers $\alpha < \beta$ there exists $q \in \mathbb{Q}$ such that $\alpha < q < \beta$ • $\mathbb{Q}$ is countable.

b. Prove closed book Proposition 4.12: The image measure μ_f is a measure on $\mathfrak{F}'$.

Written assignment 3:

Prove closed book Proposition 4.14: Every process X_t is adapted to its own filtration

Written assignment 4: Prove closed book Proposition 4.15: If τ is a random time on a filtered probability space $(\Omega, \mathfrak{F}, (\mathfrak{F}_t)_t)$, then

$$\tau \text{ is a stopping time} \quad \Leftrightarrow \quad \text{the process } (t, \omega) \mapsto X(t, \omega) := 1_{[0,\tau(\omega)[}(t) \text{ is } \mathfrak{F}_t\text{–adapted.}$$