Math 454 - Spring 2025 - Homework 05

Published: Thursday, February 13, 2025

Status - Reading Assignments:

Here are the reading assignments to be completed before the first one of this HW.

SCF2 (Shreve – Stoch. Calculus for Finance, II Textbook):
    Ch. 1 – 3.2

MF454 lecture notes:
    Ch.2 – 6

Other:
    Nothing assigned yet

New reading assignments:

In the following, MF and MF454 both refer to my course lecture notes. See the course materials page. SCF2 refers to the Shreve text (Stochastic Calculus for Finance II), WMS = Wackerly, et al: the standard Math 447 Textbook

Reading assignment 1 - due Monday, February 17:

- **a.** MF ch.7.1 (Interest Bearing Financial Assets) is remedial finance and will not be discussed in lecture. I will assume that you know that material.
- **b.** Carefully read the remainder of MF ch.7. It establishes the vocabulary for the remainder of this course and you will have problems if you are not familiar with the material.

Reading assignment 2 - due: Wednesday, February 19:

- **a.** Carefully read SCF ch.3.3 and ch.3.4 through ch.3.4.2. Skim the remainder of ch.3.4.
- **b.** Carefully read SCF ch.3.5 and skim the remainder of ch.3.

Reading assignment 3 - due Friday, February 21:

- **a.** Carefully read MF ch.8 through Proposition 8.12 in Ch.8.2. Read it slowly and work the examples and remarks!

Written assignments are on the next page.

Written assignments:

Written assignment 1: Pick about 10 formulas concerning integration from Calc 2 examples/assignments (text book) and rewrite those Riemann integrals as Lebesgue integrals, to get more familiar with the notation.

Do the same for Calc 3.

Written assignment 2: Do the fully worked examples from MF ch.4 closed book

Written assignment 3: For $f \geq 0$, how were the simple functions $f_n \uparrow f$ defined? What role do preimages play? Can you draw a picture for $\Omega = \mathbb{R}$? If you get stuck, review Step 3 of the proof of Theorem 4.16 (Integrals under Transforms) and then draw that picture.

Written assignment 4: Use the ILMD method to show the following from scratch: If Σ is the counting measure on the integers and $p : \mathbb{Z} \to [0,1];\ \omega \mapsto p(\omega)$ satisfies $\sum_{\omega \in \mathbb{Z}} p(\omega) = 1$, (i.e., $P(A) = \sum_{\omega \in A} p(\omega)$ defines a probability measure on $(\mathbb{Z}, 2^{\mathbb{Z}})$), then $\int Y dP = \sum_{\omega \in \mathbb{Z}} Y(\omega) p(\omega)$.

Written assignment 5: • How are integrals and expectations defined? • What are their properties? In particular, monotone and dominated convergence and Jensen's inequality. • What is $E[Y \mid \mathfrak{G}]$ for $\mathfrak{G} = \sigma\{G_j : j \in \mathbb{N}\}$, where $G_j \in \mathfrak{F}$ is a (countable) partition of Ω? • What is the general definition of $E[Y \mid \mathfrak{G}]$? • What are their properties? In particular, monotone and dominated convergence and Jensen's inequality.

Written assignment 6: • Why is Doob's Composition Lemma (can you state it?) a statement about "functional dependency"? • Define martingales, Markov processes, martingales. You should infer from the ch.8 readings of this homework that martingales will be extremely important in probabilistic models of financial markets.