

Math 454 - Spring 2025 - Homework 06

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Status - Reading Assignments:

Here are the reading assignments to be completed before the first one of this HW.

SCF2 (Shreve – Stoch. Calculus for Finance, II Textbook):

Ch. 1 – 3

MF454 lecture notes:

Ch.2 – 8.2, prop.8.12

Other:

Nothing assigned yet

New reading assignments:

In the following: • MF = MF454 = my course lecture notes • SCF2 = Shreve: Stochastic Calculus for Finance II

• WMS = Wackerly, et al = standard Math 447 Textbook

Reading assignment 1 - due Monday, February 24:

- a. Study for midterm 1!

Reading assignment 2 - due: Wednesday, February 26:

- a. Carefully read the remainder of MF ch.8. Skim or ignore the proof of Theorem 8.2, but be sure you understand how to do Example 8.3.

Reading assignment 3 - due Friday, February 28:

- a. Carefully read MF ch.9.1 – 9.4.
- b. Carefully read SCF2 ch.4.1 - 4.4.1

Written assignments are on the next page.

Written assignments:

Written assignment 1: Write from memory the definitions of $\nu \ll \mu$ and $\nu \sim \mu$, and prove Proposition 4.20.

Written assignment 2: Flesh out the missing details in the LMD parts of the ILMD based proof of (optional) Proposition 5.1 (Doob Composition Lemma).

Written assignment 3: Review Proposition 5.2. which shows that $X_{\mathfrak{G}} = \sum_j \frac{E[X \mathbf{1}_{G_j}]}{P(G_j)} \cdot \mathbf{1}_{G_j}$ satisfies partial averaging.

Written assignment 4: Do closed book Example 5.1.

Written assignment 5: Write from memory Definition 5.1 of $E[X | \mathfrak{G}]$ and as many properties as you remember. Then compare with Theorem 5.4 and Theorem 5.5.

Written assignment 6: Prove closed book the linearity of conditional expectations. Proof: See SCF2.

Written assignment 7: You live in a discrete time financial market with trading times at $t = 0, 1, 2$ years. The interest rate is constant, $R = 0.1$. At $t = 0$ you put \$100.00 into your bank account (asset $\mathcal{A}^{(0)}$ with portfolio holding H_t^0 , bank account price process B_t , and dollar value $V_t = B_t H_t^{(0)}$.)

What is $H_0^{(0)}$, B_0 , V_0 ? What is $H_2^{(0)}$, B_2 , V_2 ? See Ch.7.3 (The Holdings Process of a Riskless Asset).

Solution:

- $H_0^{(0)} = H_2^{(0)} = \100.00 .
- $B_0 = 1$ and $B_2 = (1 + 1/10)^2$.
- $V_0^{(0)} = B_0 \cdot H_0^{(0)} = \100.00 , and $V_2^{(0)} = B_2 \cdot H_2^{(0)} = \$(1 + 1/10)^2 \cdot 100.00$.

Written assignment 8: Write from memory the definitions of a self-financing portfolio, an arbitrage portfolio, and the discrete market budget equation.

Written assignment 9: Write from memory the value at time of expiration of a forward contract, a European call, and of a European put, where each one is agreed upon at a strike price of \$85.50, when the underlying stock has the price process S_t .