# Math 454 - Spring 2025 - Homework 14

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**Status - Reading Assignments:**

Here are the reading assignments to be completed before the first one of this HW.

SCF2 (Shreve – Stoch. Calculus for Finance, II Textbook):
Ch. 1 – 6.4.

MF454 lecture notes:
Ch.2 – ch.15.3 through Theorem 15.4

Other:
Stewart Single Variable Calculus ch.3.9 (Antiderivatives) for examples of (ordinary) differential equations.

**New reading assignments:**

In the following: • MF = MF454 = my course lecture notes • SCF2 = Shreve: Stochastic Calculus for Finance II

• WMS = Wackerly, et al = standard Math 447 Textbook

**Reading assignment 1 - due Tuesday, April 22:**

- Study for the midterm!

**Reading assignment 2 - due: Wednesday, April 23:**

- Study for the midterm!

**Reading assignment 3 - due Friday, April 18:**

**a.** Carefully read the remainder of MF ch.15.3. (Asian options.)
**b.** Read the remainder of SCF2, ch.6.

**Written assignments are on the next page.**

**Written assignments:**

**Written assignment 1:** Write from memory the definition of a $d$–dimensional brownian motion, the corresponding multiplication table for the differentials, and the Itô formula (10.8) for a function $f(t,x,y)$, where the Itô processes are driven by a twodimensional Brownian motion.

Be sure to also write from memory the differentials of those Itô processes. It does not matter that you use matching symbols, but they must be consistent!

Would you be able to repeat this exercise with a fourdimensional Brownian motion?

**Written assignment 2:** Use the multidimensional Itô formula and brute force to prove Proposition 11.1.

**Written assignment 3:** Write from memory Lévy's characterization of Brownian Motion first for dimension one, then for the multidimensional case.

**Written assignment 4:** Work line by line through the (optional) proof of Proposition 11.2. It is perfect to practice multidimensional Itô calculus.

**Written assignment 5:** Do the proof of formula (12.1) of Proposition 12.1. Repeat this exercise every three days until you can do this efficiently.

**Written assignment 6:** Remember this part about Girsanov: If an adapted process $\Theta_t$ satisfies some integrability condition, then one can construct a probability $\widetilde{P}$ equivalent to $P$ such that $d\widetilde{W}_t = \Theta_t dt + dW_t$ becomes a Brownian motion for $\widetilde{P}$. It's $d\widetilde{\vec{W}}_t = \vec{\Theta}_t dt + d\vec{W}_t$ in the multidimensional case.