Homework

Week 13.

p.554. 10.105, 106, 109, 112,

p.557. 10.115, 117, 126,

p.805. 16.1

Additional homework problems:

A1. **Homework 16.1.1.** Recall the Bayes set-up:

conditional on θ, X_1, ..., X_n are i.i.d. from $f(x|\theta)$.

Are X_i**'s i.i.d. from** f_X **?** Prove or disprove it through the assumption as follows.

$f(x|\theta)$ is the density of $bin(1,p)$, and $p \sim U(0,1)$.

A2. 448 [22] The Likelihood ratio test for H_o: $\theta \in \Theta_o$ v.s. H_a: $\theta \notin \Theta_o$ has a RR: $\{\lambda$____$k\}$, where

$\lambda =$ __________; $\hat{\theta}_o$ is the MLE under ____; $\hat{\theta}$ is the MLE under ____;

k satisfies $max\{P(RR): \ \theta \in \Theta_o\} =$ ____;

if n is large, then $-2\ln\lambda$ is approximated _______;

where $v =$ ___________; r and $r_o = \#$ of free parameters in Θ and in Θ_o, respectively.

key: $\underline{\leq}$, $\underline{\frac{L(\hat{\theta}_o)}{L(\hat{\theta})}}$, $\underline{\Theta_o}$, $\underline{\Theta}$, $\underline{\alpha}$, $\underline{\chi^2(v)}$,

24(c).

$$\begin{aligned}
\lambda = \frac{L(\hat{\theta}_o)}{L(\hat{\theta})} &= \frac{(\frac{\sum_i (x_i+y_i)}{2n})^{n\overline{x}+n\overline{y}}}{(\frac{\sum_i x_i}{n})^{n\overline{x}}(\frac{\sum_i y_i}{n})^{n\overline{y}}} & \overline{x} = 20 \ \& \ \overline{y} = 22 \\
&= \frac{21^{100(20+22)}}{20^{100(20)}22^{100(22)}} \leq k = ? & by \ 448[22] \\
-2\ln\lambda &\sim \chi^2(2-1) \\
-2\ln\lambda &= 9.53
\end{aligned}$$

H_o: $\theta_1 = \theta_2$ v.s.

H_1: $\theta_1 \neq \theta_2$

Test statistic: λ or $-2\ln\lambda$.

RR: $\lambda \leq k$? Yes, No, DNK

$-2\ln\lambda \leq \chi^2_{0.1,1}$? Yes, No, DNK

$-2\ln\lambda \geq \chi^2_{0.1,1}$? Yes, No, DNK