Exam 3 of Math 450, Fall, 2020. Name:

Part A. Fill the blanks of the formulas (30 points)
Notations in 450: $k, i, n \ge 0$ and $0 \le m \le n$. $P(X \ge 0) = 1$ and $x, t > 0$. $S_X(x) = s(x) = \Pr\{X > x\}$,

1. If $H(0) = 0$ and ____ ≥ 0, then __________ $= \int_0^\infty s(t)H'(t)\,dt$, *e.g.*,

__________ $= \int_0^\infty s(t)\,dt$, __________ $= \int_0^\infty s(t)pt^{p-1}\,dt$, $E[X \wedge a] = \int_0^{\_\_\_} s(t)\,dt$.

2. If $P(X \in \{0,1,2,...\}) =$ ____ and ____, then

$E[\_\_\_\_] = \sum_{k=1}^\infty \Pr\{X \ge k\}(H(k) - H(k-1))$, $E[\_\_\_\_] = \sum_{k=1}^\infty \Pr\{X \ge k\}$,

$E[\_\_\_\_] = \sum_{k=1}^\infty \Pr\{X \ge k\}(2k-1)$, $E[\min(X,a)] = \sum_{\_\_\_\_}^{\_\_\_\_} \Pr\{X \ge k\}$.

3. $T(x) = T_x =$ ______________, __________ $= S_{T(x)}(t) = \frac{s(x+t)}{s(x)}$, ____ $= F_{T(x)}(t) = \frac{s(x)-s(x+t)}{s(x)}$,

__________ $= \Pr\{s < T(x) \le s+t\} = {}_sp_x \cdot {}_tq_{x+s}$, ____ $= {}_{s|1}q_x$, ____ $= {}_1p_x$, ____ $= {}_1q_x$,

4. __________ $= {}_mp_x \cdot {}_np_{x+m}$, __________ $= p_xp_{x+1}\cdots p_{x+n-1}$,

__________ $= {}_{n_1}p_x \cdot {}_{n_2}p_{x+n_1} \cdot {}_{n_3}p_{x+n_1+n_2} \cdots {}_{n_k}p_{x+\sum_{j=1}^{k-1} n_j}$.

5. The force of mortality is $\mu_X(x) = \mu(x) = \mu_x =$ __________. $\mu_{T(x)}(t) =$ ________

If X is cts, ____ $= -\frac{d}{dx}\ln S_X(x)$, ____ $= \exp\left(-\int_0^x \mu(t)\,dt\right)$, ____ $= {}_tp_x\mu(x+t)$. $\mu_x(t) =$ ____

6. __________ $= E[T(x)] = \mathring{e}_{x:\overline{n}|} + {}_np_x\mathring{e}_{x+n}$, __________ $= E[T(x) \wedge n] = \mathring{e}_{x:\overline{m}|} + {}_mp_x\mathring{e}_{x+m:\overline{n-m}|}$.

7. The central rate of failure on $(x, x+n]$ is ________ $= \frac{\int_x^{x+n} S_X(t)\mu_X(t)\,dt}{\int_x^{x+n} S_X(t)\,dt} = \frac{{}_nq_x}{\mathring{e}_{x:\overline{n}|}}$,

________ $= {}_1m_x$, ________ $= E(T(x)|T(x) \le n) = \frac{\mathring{e}_{x:\overline{n}|} - n \cdot {}_np_x}{{}_nq_x}$, ________ $= {}_1a(x)$.

8. ____ $= \lceil T(x) \rceil$, $\lceil t \rceil =$ ____ if $t \in (k-1, k]$, $K(x) = K_x$____,

__________ $= {}_{k-1|}q_x = {}_{k-1}p_x \cdot q_{x+k-1} = (\prod_{j\ge0}^{k-2} p_{x+j})q_{x+k-1}$

9. ________ $= E[K(x)] = p_x(1 + e_{x+1}) = \sum_{k=1}^\infty {}_kp_x = e_{x:\overline{n}|} + {}_np_xe_{x+n}$,

________ $= E(K(x) \wedge n) = \sum_{k=1}^n {}_kp_x$, __________ $= \sum_{k=1}^\infty (2k-1) \cdot {}_kp_x$.

10. The KME $\hat{S}_{pl}(t) = \prod_{t_k \le t}$(________), and $\hat{\sigma}^2_{\hat{S}_{pl}(t)} =$ ________ $\sum_{k:\ t_k \le t} \frac{\hat{f}_{pl}(t_k)}{\hat{S}_Z(t_k-)\hat{S}_{pl}(t_k)}$.

The Nelson-Aalen estimator: $\tilde{S}_{NA}(t) =$ ____________, where $H(t) = \sum_{t_k \le t} \frac{d_k}{r_k}$.

$\hat{\sigma}^2_{\tilde{S}_{NA}(t)} = (\tilde{S}_{NA}(t))^2\hat{\sigma}^2_{H(t)}$, where $\hat{\sigma}^2_{H(t)} =$ ____________.

11. ____________ $=$ **# of individuals alive** at age x, ____________ $= L_x$,

${}_td_x =$ ____________, $d_x =$ ____________, $s(x) =$ ____________, ${}_tp_x =$ ______,

______ $= \prod_{x \le k < x+t}(1 - d_k/l_k)$. ________ $= \ell_x\mathring{e}_x = \int_0^\infty \ell_{x+t}\,dt = \sum_{k \ge x} L_k$
= E(# of years lived beyong age x by the cohort group with l_0 members),

__________ $= \ell_x\mathring{e}_{x:\overline{n}|} = T_x - T_{x+n}$, ($T_x$ in #3 differs from T_x in #11)

12.

Interpolation	ℓ_{x+t}	${}_tp_x$
UDD		
exponential		
Balducci		

, where ______.

13. **Application of Woolhouse's formula:**

$$\overline{a}_x \approx \underline{\qquad\qquad\qquad} - \frac{1}{12}(\delta + \mu(x)) \approx \underline{\qquad\qquad\qquad} - \frac{1}{12m^2}(\delta + \mu(x))$$

14. Life Insurance $Z = \underline{\qquad\qquad}$

Whole life ins: $Z_x = \underline{\qquad\qquad}$, $\overline{Z}_x = \underline{\qquad\qquad}$, $A_x = A_x(v) = \underline{\qquad\qquad} = \underline{\qquad\qquad\qquad}$,

${}^2A_x = E[Z_x^2] = \underline{\qquad\qquad}$,

n–year term : $Z^1_{x:\overline{n}|} = \underline{\qquad\qquad}$, $\overline{Z}^1_{x:\overline{n}|} = \underline{\qquad\qquad}$, $A^1_{x:\overline{n}|} = E[Z^1_{x:\overline{n}|}] = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad}$,

${}^2A^1_{x:\overline{n}|} = E((Z^1_{x:\overline{n}|})^2) = \underline{\qquad\qquad}$,

n–year deferred : ${}_n|Z_x = \underline{\qquad\qquad\qquad\qquad}$, ${}_n|\overline{Z}_x = \underline{\qquad\qquad\qquad\qquad}$,

${}^2{}_n|A_x = {}_n|A_x$ ______, ${}_n|A_x = E[{}_n|Z_x] = \underline{\qquad\qquad}$ $v^k f_{K_x}(k) = \underline{\qquad\qquad\qquad\qquad\qquad} = vp_x \cdot {}_{n-1}|A_{x+1}$.

n–year pure endowment : $Z_{x:\overset{1}{\overline{n}|}} = \underline{\qquad\qquad}$, $\overline{Z}_{x:\overset{1}{\overline{n}|}} = \underline{\qquad\qquad\qquad}$,
$A_{x:\overset{1}{\overline{n}|}} = {}_nE_x = \underline{\qquad\qquad\qquad}$, ${}^2A_{x:\overset{1}{\overline{n}|}} = E((Z_{x:\overset{1}{\overline{n}|}})^2) = A_{x:\overset{1}{\overline{n}|}}\underline{\qquad\qquad}$,

n–year endowment : $Z_{x:\overline{n}|} = \underline{\qquad\qquad}$, $\overline{Z}_{x:\overline{n}|} = \underline{\qquad\qquad}$, $A_{x:\overline{n}|} = \underline{\qquad\qquad\qquad\qquad\qquad}$,
${}^2A_{x:\overline{n}|} = E((Z_{x:\overline{n}|})^2) = A_{x:\overline{n}|}\underline{\qquad\quad}$,
m–year defer n-year term : ${}_m|{}_nZ_x = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$, ${}_m|{}_n\overline{Z}_x = \underline{\qquad\qquad\qquad\qquad\qquad}$,

${}_m|{}_nA_x = {}_mE_x\underline{\qquad\qquad\qquad\qquad}$, ${}^2{}_m|{}_nA_{x:\overline{n}|} = E[({}_m|{}_nZ_x)^2] = \underline{\qquad\qquad\qquad}$ $v^{2k}f_{K_x}(k)$,

$\underline{\qquad\qquad} = Z^1_{x:\overline{n}|} + {}_n|Z_x$, $Z^1_{x:\overline{n}|} \cdot {}_n|Z_x = \underline{\qquad\quad}$, $\underline{\qquad\qquad\qquad} = Z^1_{x:\overline{n}|} + Z_{x:\overset{1}{\overline{n}|}}$, $Z^1_{x:\overline{n}|} \cdot Z_{x:\overset{1}{\overline{n}|}} = \underline{\qquad\quad}$,

$\underline{\qquad\qquad\qquad} = {}_nE_xA_{x+n}$, $\underline{\qquad\qquad} = A^1_{x:\overline{n}|} + {}_nE_xA_{x+n}$.

15. $(IZ)_x = \underline{\qquad\quad}$. $(\overline{IZ})_x = \underline{\qquad\quad}$. $(I\overline{Z})_x = \underline{\qquad\quad}$. $(DZ)^1_{x:\overline{n}|} = \underline{\qquad\qquad\qquad\qquad\qquad}$.

$(\overline{DZ})^1_{x:\overline{n}|} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$. $(D\overline{Z})^1_{x:\overline{n}|} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$.

16. $\underline{\qquad\quad} = \frac{1}{i+1}$, $\underline{\qquad\quad} = -\ln v$, $\underline{\qquad\quad} = 1 - v$. $\sum_{k=1}^n kx^{k-1} = \underline{\qquad\qquad\quad}$ $a_{\overline{n}|} = \sum_{k=1}^n v^k = \underline{\qquad\quad}$

$\sum_{k=0}^n v^k = \underline{\qquad\quad}$.

due	PV $\quad \ddot{a}_{\overline{n}|} = \_\_\_\_\_\_ = \_\_\_\_\_\_$	APV
whole life	$\ddot{Y}_x = \sum_{k=0}^{K_x-1} v^k = \_\_\_\_\_\_$	$\ddot{a}_x = \_\_\_\_\_\_$
n–y. def.	${}_n\vert\ddot{Y}_x = \_\_\_\_\_\_ = \frac{v^n - v^{K_x}}{1-v} \_\_\_\_\_\_$	${}_n\vert\ddot{a}_x = \_\_\_\_\_\_$
n–y. temp.	$\ddot{Y}_{x:\overline{n}\vert} = \_\_\_\_\_\_ = \frac{1-Z_{x:\overline{n}\vert}}{d}$	$\ddot{a}_{x:\overline{n}\vert} = \_\_\_\_\_\_$
n–y. cer.	$\ddot{Y}_{\overline{x:\overline{n}\vert}} = \_\_\_\_\_\_ = \ddot{a}_{\overline{n}\vert} + {}_n\vert\ddot{Y}_x$	$\ddot{a}_{\overline{x:\overline{n}\vert}} = \_\_\_\_\_\_$

17. $i \neq 0$.

immediate: $Y_x = \sum_{k\geq 1}^{K_x-1} v^k = \_\_\_\_\_\_$, $\quad {}_n|Y_x = \sum_{k>n}^{K_x-1} v^k = \_\_\_\_\_\_$, $\quad Y_{x:\overline{n}|} = \sum_{k=1}^{(K_x-1)\wedge n} v^k = \_\_\_\_\_\_$, $\quad Y_{\overline{x:\overline{n}|}} = \sum_{k=1}^{(K_x-1)\vee n} v^k = \_\_\_\_\_\_$

cts	present value$=\bar{a}_{\overline{n}\vert} = ______$	APV
whole life	$\overline{Y}_x = \int_0^{T_x} v^t dt = \_\_\_\_\_\_$	$\bar{a}_x = \_\_\_\_\_\_ dt$
n–y. def.	${}_n\vert\overline{Y}_x = \int_n^{T_x} v^t dt \_\_\_\_\_\_$	${}_n\vert\bar{a}_x = \_\_\_\_\_\_ dt$
n–y. temp.	$\overline{Y}_{x:\overline{n}\vert} = \int_0^{T_x\wedge n} v^t dt = \_\_\_\_\_\_$	$\bar{a}_{x:\overline{n}\vert} = \_\_\_\_\_\_ dt$
n–y. cer.	$\overline{Y}_{\overline{x:\overline{n}\vert}} = \int_0^{n\vee T_x} v^t dt = \_\_\_\_\_\_$	$a_{\overline{x:\overline{n}\vert}} = \_\_\_\_\_\_$

18. $\_\_\_\_ = \ddot{Y}_{x:\overline{n}|} + {}_n|\ddot{Y}_x$. $\quad E((\ddot{Y}_x)^2)\_\_\_\_\ddot{a}_x(v^2)$. $\quad \ddot{a}_x = \_\_\_\_\_\_$.

$\bar{a}_x = \_\_\_\_\_\_$. $\quad {}_n|\ddot{a}_x = {}_nE_x\ddot{a}_{x+n} = \_\_\_\_\_\_$.

${}_n|\bar{a}_x = {}_nE_x \cdot \bar{a}_{x+n} = \_\_\_\_\_\_$. $\quad \ddot{a}_{x:\overline{n+m}|} = \ddot{a}_{x:\overline{n}|}\_\_\_\_\_\_$.