Notations in 450: $k, i, n \geq 0$ and $0 \leq m \leq n$. $P(X \geq 0) = 1$ and $x, t > 0$. $S_X(x) = s(x) = \mathbb{P}\{X > x\}$,

1. If $H(0) = 0$ and $H' \geq 0$, then $E(H(X)) = \int_0^\infty s(t)H'(t)\,dt$, *e.g.*,
   $E(X) = \int_0^\infty s(t)\,dt$, $E(X^p) = \int_0^\infty s(t)pt^{p-1}\,dt$, $E[X \wedge a] = \int_0^a s(t)\,dt$.
2. If $P(X \in \{0, 1, 2, ...\}) = 1$ and $H \uparrow$, then
   $E[H(X)] = \sum_{k=1}^\infty \mathbb{P}\{X \geq k\}(H(k) - H(k-1))$, $E[X] = \sum_{k=1}^\infty \mathbb{P}\{X \geq k\}$,
   $E[X^2] = \sum_{k=1}^\infty \mathbb{P}\{X \geq k\}(2k-1)$, $E[\min(X, a)] = \sum_{k=1}^a \mathbb{P}\{X \geq k\}$.
3. $T(x) = T_x = (X - x)|(X > x)$,
   ${}_tp_x = S_{T(x)}(t) = \frac{s(x+t)}{s(x)}$, ${}_tq_x = F_{T(x)}(t) = \frac{s(x)-s(x+t)}{s(x)}$,
   ${}_{s|t}q_x = \mathbb{P}\{s < T(x) \leq s+t\} = {}_sp_x \cdot {}_tq_{x+s}$, ${}_{s|}q_x = {}_{s|1}q_x$, $p_x = {}_1p_x$, $q_x = {}_1q_x$,
4. ${}_{m+n}p_x = {}_mp_x \cdot {}_np_{x+m}$, ${}_np_x = p_x p_{x+1} \cdots p_{x+n-1}$,
   ${}_{\sum_{j=1}^k n_j}p_x = {}_{n_1}p_x \cdot {}_{n_2}p_{x+n_1} \cdot {}_{n_3}p_{x+n_1+n_2} \cdots {}_{n_k}p_{x+\sum_{j=1}^{k-1} n_j}$.
5. The force of mortality is $\mu_X(x) = \mu(x) = \mu_x = \frac{f_X(x)}{S_X(x-)}$. $\mu_{T(x)}(t) = \mu_x(t)$. If X is cts,
   $\mu(x) = -\frac{d}{dx}\ln S_X(x)$, $S_X(x) = \exp\left(-\int_0^x \mu(t)\,dt\right)$, $f_{T(x)}(t) = {}_tp_x\mu(x+t)$. $\mu_x(t) = \mu(x+t)$.
6. $\mathring{e}_x = E[T(x)] = \mathring{e}_{x:\overline{n}|} + {}_np_x\mathring{e}_{x+n}$, $\mathring{e}_{x:\overline{n}|} = E[T(x) \wedge n] = \mathring{e}_{x:\overline{m}|} + {}_mp_x\mathring{e}_{x+m:\overline{n-m}|}$.
7. The central rate of failure on $(x, x+n]$ is ${}_nm_x = \frac{\int_0^n {}_tp_x\mu_x(t)\,dt}{\int_0^n {}_tp_x\,dt} = \frac{{}_nq_x}{\mathring{e}_{x:\overline{n}|}}$,
   $m_x = {}_1m_x$, ${}_na(x) = E(T(x)|T(x) \leq n) = \frac{\mathring{e}_{x:\overline{n}|} - n \cdot {}_np_x}{{}_nq_x}$, $a(x) = {}_1a(x)$.
8. $K_x = \lceil T(x) \rceil$, $\lceil t \rceil = k$ if $t \in (k-1, k]$, $K(x) = K_x - 1$,
   $f_{K_x}(k) = {}_{k-1|}q_x = {}_{k-1}p_x \cdot q_{x+k-1} = (\prod_{j\geq 0}^{k-2} p_{x+j})q_{x+k-1}$
9. $e_x = E[K(x)] = p_x(1 + e_{x+1}) = \sum_{k=1}^\infty {}_kp_x = e_{x:\overline{n}|} + {}_np_x e_{x+n}$,
   $e_{x:\overline{n}|} = E(K(x) \wedge n) = \sum_{k=1}^n {}_kp_x$, $E[(K(x))^2] = \sum_{k=1}^\infty (2k-1) \cdot {}_kp_x$.
10. The KME $\hat{S}_{pl}(t) = \prod_{t_k \leq t}(1 - \frac{d_k}{r_k})$, and $\hat{\sigma}^2_{\hat{S}_{pl}(t)} = \frac{1}{n}(\hat{S}_{pl}(t))^2 \sum_{k:\ t_k \leq t} \frac{\hat{f}_{pl}(t_k)}{\hat{S}_Z(t_k-)\hat{S}_{pl}(t_k)}$.
    The Nelson-Aalen estimator: $\tilde{S}_{NA}(t) = e^{-H(t)}$, where $H(t) = \sum_{t_k \leq t} \frac{d_k}{r_k}$.
    $\hat{\sigma}^2_{\tilde{S}_{NA}(t)} = (\tilde{S}_{NA}(t))^2 \hat{\sigma}^2_{H(t)}$, where $\hat{\sigma}^2_{H(t)} = \sum_{t_j \leq t} \frac{(r_j - d_j)d_j}{(r_j-1)r_j^2}$.
11. $\ell_x = \#$ of individuals alive at age x, ${}_1L_x = L_x$. ${}_td_x = \ell_x - \ell_{x+t}$, $d_x = {}_1d_x$,
    ${}_tp_x = \prod_{x \leq k < x+t}(1 - d_k/l_k)$. $T_x = \ell_x\mathring{e}_x = \int_0^\infty \ell_{x+t}\,dt = \sum_{k=x}^\infty L_k$
    = E(# of years lived beyong age x by the cohort group with l_0 members),
    ${}_nL_x = \ell_x\mathring{e}_{x:\overline{n}|} = T_x - T_{x+n}$. $s(x) = \frac{\ell_x}{\ell_0}$, ${}_tp_x = \frac{\ell_{x+t}}{\ell_x}$, ($T_x$ in #3 differs from T_x in #11).
12.

Interpolation	ℓ_{x+t}	${}_tp_x$
UDD		
exponential		
Balducci		

where _____.

key:

UDD	$(1-t)\ell_x + t\ell_{x+1}$ or $\ell_x - td_x$	$1 - tq_x$
exponential	$(\ell_x)^{1-t}(\ell_{x+1})^t$ or $\ell_x p_x^t$	p_x^t
Balducci	$\frac{1}{(1-t)\frac{1}{\ell_x} + t\frac{1}{\ell_{x+1}}}$	$\frac{p_x}{t+(1-t)p_x}$

, $t \in [0, 1]$.

14. **Life Insurance:** $Z = b_{T_x} v_{T_x}$,
Whole life ins: $Z_x = v^{K_x}$, $\overline{Z}_x = v^{T_x}$, $A_x = A_x(v) = E[v^{K_x}] = vq_x + vp_x A_{x+1}$, ${}^2A_x = E[Z_x^2] = A_x(v^2)$.

n–year term : $Z^1_{x:\overline{n}|} = v^{K_x} I(K_x \le n)$, $\overline{Z}^1_{x:\overline{n}|} = v^{T_x} I(T_x \le n)$, $A^1_{x:\overline{n}|} = E[Z^1_{x:\overline{n}|}] = \sum_{k=1}^{n} v^k f_{K_x}(k) = vq_x + vp_x A^1_{x+1:\overline{n-1}|}$, ${}^2A^1_{x:\overline{n}|} = E((Z^1_{x:\overline{n}|})^2) = A^1_{x:\overline{n}|}(v^2)$.

n–year deferred : ${}_{n}|Z_x = v^{K_x} I(n < K_x)$, ${}_{n}|\overline{Z}_x = v^{T_x} I(n < T_x)$, ${}^2{}_{n}|A_x = {}_{n}|A_x\ (v^2)$, ${}_{n}|A_x = E[{}_{n}|Z_x] = \sum_{k=n+1}^{\infty} v^k f_{K_x}(k) = v^{n+1} f_{K_x}(n+1) + {}_{n+1}|A_x = vp_x \cdot {}_{n-1}|A_{x+1}$.

n–year pure endowment : $Z_{x:\overline{n}|}^{\ \ 1} = v^n I(n < K_x)$, $\overline{Z}_{x:\overline{n}|}^{\ \ 1} = v^n I(n < T_x)$, $A_{x:\overline{n}|}^{\ \ 1} = {}_nE_x = v^n\, {}_np_x$, ${}^2A_{x:\overline{n}|}^{\ \ 1} = E((Z_{x:\overline{n}|}^{\ \ 1})^2) = A_{x:\overline{n}|}^{\ \ 1}(v^2)$,

n–year endowment : $Z_{x:\overline{n}|} = v^{K_x \wedge n}$, $\overline{Z}_{x:\overline{n}|} = v^{T_x \wedge n}$, $A_{x:\overline{n}|} = \sum_{k=1}^{n} v^k f_{K_x}(k) + v^n\, {}_np_x$, ${}^2A_{x:\overline{n}|} = E((Z_{x:\overline{n}|})^2) = A_{x:\overline{n}|}(v^2)$,

m–year defer n-year term : ${}_{m}|_{n}Z_x = v^{K_x} I(m < K_x \le n+m)$, ${}_{m}|_{n}\overline{Z}_x = v^{T_x} I(m < T_x \le n+m)$, ${}_{m}|_{n}A_x = {}_mE_x A^1_{x+m:\overline{n}|}$, ${}^2{}_{m}|_{n}A_{x:\overline{n}|} = E[({}_{m}|_{n}Z_x)^2] = \sum_{k=m+1}^{m+n} v^{2k} f_{K_x}(k)$.

$Z_x = Z^1_{x:\overline{n}|} + {}_{n}|Z_x$, $\quad Z^1_{x:\overline{n}|} \cdot {}_{n}|Z_x = 0$,
$Z_{x:\overline{n}|} = Z^1_{x:\overline{n}|} + Z_{x:\overline{n}|}^{\ \ 1}$, $Z^1_{x:\overline{n}|} \cdot Z_{x:\overline{n}|}^{\ \ 1} = 0$,
${}_{n}|A_x = {}_nE_x A_{x+n}$, $\quad A_x = A^1_{x:\overline{n}|} + {}_nE_x A_{x+n}$.
15. $(IZ)_x = K_x v^{K_x}$. $(\overline{IZ})_x = T_x v^{T_x}$. $(I\overline{Z})_x = \lceil T_x \rceil v^{T_x}$. $(DZ)^1_{x:\overline{n}|} = (n+1-K_x) v^{K_x} I(K_x \le n)$. $(\overline{DZ})^1_{x:\overline{n}|} = (n - T_x) v^{T_x} I(T_x \le n)$. $(D\overline{Z})^1_{x:\overline{n}|} = \lceil n - T_x \rceil v^{T_x} I(T_x \le n)$.
16. $v = \frac{1}{i+1}$, $\delta = -\ln v$, $d = 1 - v$. $\sum_{k=1}^{n} k x^{k-1} = (\frac{1-x^{n+1}}{1-x})'_x$.

$a_{\overline{n}|} = \sum_{k=1}^{n} v^k = v(\frac{1-v^n}{1-v})$. $\quad \sum_{k=0}^{n} v^k = \frac{1-v^{n+1}}{1-v}$.

17.

due	PV $\quad \ddot{a}_{\overline{n}|} = \sum_{k=0}^{n-1} v^k = \frac{1-v^n}{1-v}$	APV
whole life	$\ddot{Y}_x = \sum_{k=0}^{K_x-1} v^k = \frac{1-Z_x}{1-v}$	$\ddot{a}_x = \sum_{k=0}^{\infty} v^k\, {}_kp_x$
n–y. def.	${}_{n}|\ddot{Y}_x = \sum_{k\ge n}^{K_x-1} v^k = \frac{v^n - v^{K_x}}{1-v} I(K_x > n)$	${}_{n}|\ddot{a}_x = \sum_{k=n}^{\infty} v^k\, {}_kp_x$
n–y. tem.	$\ddot{Y}_{x:\overline{n}|} = \sum_{k=0}^{(K_x \wedge n)-1} v^k = \frac{1-Z_{x:\overline{n}|}}{d}$	$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k\, {}_kp_x$
n–y. cer.	$\ddot{Y}_{\overline{x:\overline{n}|}} = \sum_{k=0}^{(n \vee K_x)-1} v^k = \ddot{a}_{\overline{n}|} + {}_{n}|\ddot{Y}_x$	$\ddot{a}_{\overline{x:\overline{n}|}} = \ddot{a}_{\overline{n}|} + {}_{n}|\ddot{a}_x$

$i \ne 0$.

immediate: $Y_x = \sum_{k\ge 1}^{K_x-1} v^k = \ddot{Y}_x - 1$, $\quad {}_{n}|Y_x = \sum_{k>n}^{K_x-1} v^k = {}_{n+1}|\ddot{Y}_x$,
$Y_{x:\overline{n}|} = \sum_{k=1}^{(K_x-1)\wedge n} v^k = \ddot{Y}_{x:\overline{n+1}|} - 1$, $\quad Y_{\overline{x:\overline{n}|}} = \sum_{k=1}^{(K_x-1)\vee n} v^k = \ddot{Y}_{\overline{x:\overline{n+1}|}} - 1$

cts	present value=$\overline{a}_{\overline{n}|} = \underline{\int_0^n v^t dt}$	APV
whole life	$\overline{Y}_x = \int_0^{T_x} v^t dt = \underline{\frac{1-\overline{Z}_x}{\delta}}$	$\overline{a}_x = \underline{\int_0^\infty v^t {}_tp_x dt}$
n–y. def.	${}_n|\overline{Y}_x = \int_n^{T_x} v^t dt \underline{I(T_x > n)}$	${}_n|\overline{a}_x = \underline{\int_n^\infty v^t {}_tp_x dt}$
n–y. tem.	$\overline{Y}_{x:\overline{n}|} = \int_0^{T_x \wedge n} v^t dt = \underline{\frac{1-v^{T_x \wedge n}}{\delta}}$	$\overline{a}_{x:\overline{n}|} = \underline{\int_0^n v^t {}_tp_x dt}$
n–y. cer.	$\overline{Y}_{\overline{x:\overline{n}|}} = \int_0^{T_x \vee n} v^t dt = \underline{\overline{a}_{\overline{n}|} + {}_n|\overline{Y}_x}$	$\overline{a}_{\overline{x:\overline{n}|}} = \underline{\overline{a}_{\overline{n}|} + {}_n|\overline{a}_x}$

18. $\underline{\ddot{Y}_x} = \ddot{Y}_{x:\overline{n}|} + {}_n|\ddot{Y}_x$, $\quad E((\ddot{Y}_x)^2)\underline{\neq}\ddot{a}_x(v^2)$, $\ddot{a}_x = \underline{1 + vp_x\ddot{a}_{x+1}}$. $\quad \overline{a}_x = \underline{\overline{a}_{x:\overline{n}|} + v^n {}_np_x\overline{a}_{x+n}}$.
${}_n|\ddot{a}_x = {}_nE_x\ddot{a}_{x+n} = \underline{vp_x \cdot {}_{n-1}|\ddot{a}_{x+1}}$. $\quad {}_n|\overline{a}_x = {}_nE_x \cdot \overline{a}_{x+n} = \underline{vp_x \cdot {}_{n-1}|\overline{a}_{x+1}}$.
$\ddot{a}_{x:\overline{n+m}|} = \ddot{a}_{x:\overline{n}|}\underline{+{}_nE_x \cdot \ddot{a}_{x+n:\overline{m}|}}$.

19.

Plan	Loss
Whole life insurance	$\underline{Z_x - P\ddot{Y}_x}$
t–year funded whole life insurance	$\underline{Z_x - P\ddot{Y}_{x:\overline{t}|}}$
n–year term insurance	$\underline{Z^1_{x:\overline{n}|} - P\ddot{Y}_{x:\overline{n}|}}$
t–year funded n–year term insurance	$\underline{Z^1_{x:\overline{n}|} - P\ddot{Y}_{x:\overline{t}|}}$
n–year pure endowment insurance	$\underline{Z_{x:\overset{1}{\overline{n}|}} - P\ddot{Y}_{x:\overline{n}|}}$
t–year funded n–year pure endowment insurance	$\underline{Z_{x:\overset{1}{\overline{n}|}} - P\ddot{Y}_{x:\overline{t}|}}$
n–year endowment	$\underline{Z_{x:\overline{n}|} - P\ddot{Y}_{x:\overline{n}|}}$
t–year funded n–year endowment insurance	$\underline{Z_{x:\overline{n}|} - P\ddot{Y}_{x:\overline{t}}|}$
n–year deferred insurance	$\underline{{}_n|Z_x - P\ddot{Y}_x}$
t–year funded n–year deferred insurance	$\underline{{}_n|Z_x - P\ddot{Y}_{x:\overline{t}|}}$

Loss in the fully discrete case

13. **Application of Woolhouse's formula:**

$$\overline{a}_x \approx \underline{\ddot{a}_x - \frac{1}{2}} - \frac{1}{12}(\delta + \mu_x) \approx \underline{\ddot{a}_x^{(m)} - \frac{1}{2m}} - \frac{1}{12m^2}(\delta + \mu_x)$$

Notations in 450: $k, i, n \geq 0$ and $0 \leq m \leq n$. $P(X \geq 0) = 1$ and $x, t > 0$. $S_X(x) = s(x) = \mathbb{P}\{X > x\}$,

1. If $H(0) = 0$ and _____ ≥ 0, then __________ $= \int_0^\infty s(t)H'(t)\,dt$, *e.g.*,

   __________ $= \int_0^\infty s(t)\,dt$, __________ $= \int_0^\infty s(t)pt^{p-1}\,dt$, $E[X \wedge a] = \int_0^{\_\_\_} s(t)\,dt$.

2. If $P(X \in \{0, 1, 2, ...\}) =$ _____ and _____, then

   $E[\_\_\_\_\_] = \sum_{k=1}^\infty \mathbb{P}\{X \geq k\}(H(k) - H(k-1))$, $E[\_\_\_\_\_] = \sum_{k=1}^\infty \mathbb{P}\{X \geq k\}$,

   $E[\_\_\_\_\_] = \sum_{k=1}^\infty \mathbb{P}\{X \geq k\}(2k-1)$, $E[\min(X, a)] = \sum_{\_\_\_\_\_}^{\_\_\_\_\_} \mathbb{P}\{X \geq k\}$.

3. $T(x) = T_x =$ _______________, ___________ $= S_{T(x)}(t) = \frac{s(x+t)}{s(x)}$, _____ $= F_{T(x)}(t) = \frac{s(x)-s(x+t)}{s(x)}$,

   __________ $= \mathbb{P}\{s < T(x) \leq s+t\} = {}_sp_x \cdot {}_tq_{x+s}$, _____ $= {}_{s|1}q_x$, _____ $= {}_1p_x$, _____ $= {}_1q_x$,

4. __________ $= {}_mp_x \cdot {}_np_{x+m}$, __________ $= p_xp_{x+1} \cdots p_{x+n-1}$,

   __________ $= {}_{n_1}p_x \cdot {}_{n_2}p_{x+n_1} \cdot {}_{n_3}p_{x+n_1+n_2} \cdots {}_{n_k}p_{x+\sum_{j=1}^{k-1} n_j}$.

5. The force of mortality is $\mu_X(x) = \mu(x) = \mu_x =$ __________. $\mu_{T(x)}(t) =$ ________

   If X is cts, _____ $= -\frac{d}{dx}\ln S_X(x)$, _____ $= \exp\left(-\int_0^x \mu(t)\,dt\right)$, _____ $= {}_tp_x\mu(x+t)$. $\mu_x(t) =$ _____

6. __________ $= E[T(x)] = \mathring{e}_{x:\overline{n}|} + {}_np_x\mathring{e}_{x+n}$, __________ $= E[T(x) \wedge n] = \mathring{e}_{x:\overline{m}|} + {}_mp_x\mathring{e}_{x+m:\overline{n-m}|}$.

7. The central rate of failure on $(x, x+n]$ is ________ $= \frac{\int_0^n {}_tp_x\mu_x(t)\,dt}{\int_0^n {}_tp_x\,dt} = \frac{{}_nq_x}{\mathring{e}_{x:\overline{n}|}}$,

   ________ $= {}_1m_x$, ________ $= E(T(x)|T(x) \leq n) = \frac{\mathring{e}_{x:\overline{n}|} - n \cdot {}_np_x}{{}_nq_x}$, ________ $= {}_1a(x)$.

8. _____ $= \lceil T(x) \rceil$, $\lceil t \rceil =$ _____ if $t \in (k-1, k]$, $K(x) = K_x$_____,

   __________ $= {}_{k-1|}q_x = {}_{k-1}p_x \cdot q_{x+k-1} = (\prod_{j\geq 0}^{k-2} p_{x+j})q_{x+k-1}$

9. ________ $= E[K(x)] = p_x(1 + e_{x+1}) = \sum_{k=1}^\infty {}_kp_x = e_{x:\overline{n}|} + {}_np_xe_{x+n}$,

$\_\_\_\_\_\_ = E(K(x) \wedge n) = \sum_{k=1}^{n} {}_kp_x$, $\_\_\_\_\_\_ = \sum_{k=1}^{\infty}(2k-1) \cdot {}_kp_x$.

10. The KME $\hat{S}_{pl}(t) = \prod_{t_k \le t}(\_\_\_\_\_\_)$, and $\hat{\sigma}^2_{\hat{S}_{pl}(t)} = \_\_\_\_\_\_ \sum_{k:\ t_k \le t} \frac{\hat{f}_{pl}(t_k)}{\hat{S}_Z(t_k-)\hat{S}_{pl}(t_k)}$.

The Nelson-Aalen estimator: $\tilde{S}_{NA}(t) = \_\_\_\_\_\_$, where $H(t) = \sum_{t_k \le t} \frac{d_k}{r_k}$.

$\hat{\sigma}^2_{\tilde{S}_{NA}(t)} = (\tilde{S}_{NA}(t))^2 \hat{\sigma}^2_{H(t)}$, where $\hat{\sigma}^2_{H(t)} = \_\_\_\_\_\_$.

11. $\_\_\_\_\_\_$ = **# of individuals alive** at age x, $\_\_\_\_\_\_ = L_x$,

${}_td_x = \_\_\_\_\_\_$, $d_x = \_\_\_\_\_\_$, $s(x) = \_\_\_\_\_\_$, ${}_tp_x = \_\_\_\_\_\_$,

$\_\_\_\_\_\_ = \prod_{x \le k < x+t}(1 - d_k/l_k)$. $\_\_\_\_\_\_ = \ell_x \mathring{e}_x = \int_0^{\infty} \ell_{x+t}\,dt = \sum_{k \ge x} L_k$
= E(# of years lived beyong age x by the cohort group with l_0 members),

$\_\_\_\_\_\_ = \ell_x \mathring{e}_{x:\overline{n}|} = T_x - T_{x+n}$, ($T_x$ in #3 differs from T_x in #11)

12.

Interpolation	ℓ_{x+t}	${}_tp_x$
UDD		
exponential		
Balducci		

, where _____.

13. **Application of Woolhouse's formula:**

$$\overline{a}_x \approx \_\_\_\_\_\_ - \frac{1}{12}(\delta + \mu(x)) \approx \_\_\_\_\_\_ - \frac{1}{12m^2}(\delta + \mu(x))$$

14. Life Insurance $Z = \_\_\_\_\_\_$

Whole life ins: $Z_x = \_\_\_\_\_\_$, $\overline{Z}_x = \_\_\_\_\_\_$, $A_x = A_x(v) = \_\_\_\_\_\_ = \_\_\_\_\_\_$,

${}^2A_x = E[Z_x^2] = \_\_\_\_\_\_$,

n–year term : $Z^1_{x:\overline{n}|} = \_\_\_\_\_\_$, $\overline{Z}^1_{x:\overline{n}|} = \_\_\_\_\_\_$, $A^1_{x:\overline{n}|} = E[Z^1_{x:\overline{n}|}] = \_\_\_\_\_\_ = \_\_\_\_\_\_$,

${}^2A^1_{x:\overline{n}|} = E((Z^1_{x:\overline{n}|})^2) = \_\_\_\_\_\_$,

n–year deferred : ${}_{n|}Z_x = \_\_\_\_\_\_$, ${}_{n|}\overline{Z}_x = \_\_\_\_\_\_$,

$^2{}_{n|}A_x = {}_{n|}A_x$ ______, ${}_{n|}A_x = E[{}_{n|}Z_x] =$ ________ $v^k f_{K_x}(k) =$ ____________________ $= vp_x \cdot {}_{n-1|}A_{x+1}$.

n–year pure endowment : $Z_{x:\overline{n}|}^{\ \ 1} =$ ________ , $\overline{Z}_{x:\overline{n}|}^{\ \ 1} =$ __________,
$A_{x:\overline{n}|}^{\ \ 1} = {}_nE_x =$ ___________, $^2A_{x:\overline{n}|}^{\ \ 1} = E((Z_{x:\overline{n}|}^{\ \ 1})^2) = A_{x:\overline{n}|}^{\ \ 1}$________,

n–year endowment : $Z_{x:\overline{n}|} =$ _________, $\overline{Z}_{x:\overline{n}|} =$ _________, $A_{x:\overline{n}|} =$ ___________________,

$^2A_{x:\overline{n}|} = E((Z_{x:\overline{n}|})^2) = A_{x:\overline{n}|}$______,
m–year defer n-year term : ${}_{m|n}Z_x =$ ________________________, ${}_{m|n}\overline{Z}_x =$ _____________________,

${}_{m|n}A_x = {}_mE_x$_______________, $^2{}_{m|n}A_{x:\overline{n}|} = E[({}_{m|n}Z_x)^2] =$ __________ $v^{2k}f_{K_x}(k)$,

________ $= Z^1_{x:\overline{n}|} + {}_{n|}Z_x$, $Z^1_{x:\overline{n}|} \cdot {}_{n|}Z_x =$ ______, ____________ $= Z^1_{x:\overline{n}|} + Z_{x:\overline{n}|}^{\ \ 1}$, $Z^1_{x:\overline{n}|} \cdot Z_{x:\overline{n}|}^{\ \ 1} =$ ______,

____________ $= {}_nE_xA_{x+n}$, ________ $= A^1_{x:\overline{n}|} + {}_nE_xA_{x+n}$.

15. $(IZ)_x =$ _____. $(\overline{IZ})_x =$ _____. $(I\overline{Z})_x =$ _____. $(DZ)^1_{x:\overline{n}|} =$ __________________.

$(\overline{DZ})^1_{x:\overline{n}|} =$ __________________. $(D\overline{Z})^1_{x:\overline{n}|} =$ __________________.

16. _____ $= \frac{1}{i+1}$, _____ $= -\ln v$, _____ $= 1 - v$. $\sum_{k=1}^n kx^{k-1} =$ _________ $a_{\overline{n}|} = \sum_{k=1}^n v^k$ $=$ _____

$\sum_{k=0}^n v^k =$ _____.

17.

due	PV $\ddot{a}_{\overline{n}\vert} =$ _________ $=$ _________	APV
whole life	$\ddot{Y}_x = \sum_{k=0}^{K_x-1} v^k =$ ____________	$\ddot{a}_x =$ _____________
n–y. def.	${}_{n\vert}\ddot{Y}_x =$ _____________ $= \frac{v^n - v^{K_x}}{1-v}$_____________	${}_{n\vert}\ddot{a}_x =$ _____________
n–y. temp.	$\ddot{Y}_{x:\overline{n}\vert} =$ _____________ $= \frac{1-Z_{x:\overline{n}\vert}}{d}$	$\ddot{a}_{x:\overline{n}\vert} =$ _____________
n–y. cer.	$\ddot{Y}_{\overline{x:\overline{n}\vert}} =$ _____________ $= \ddot{a}_{\overline{n}\vert} + {}_{n\vert}\ddot{Y}_x$	$\ddot{a}_{\overline{x:\overline{n}\vert}} =$ _____________

$i \neq 0$.

immediate: $Y_x = \sum_{k\geq 1}^{K_x-1} v^k =$ __________, ${}_n|Y_x = \sum_{k>n}^{K_x-1} v^k =$ __________, $Y_{x:\overline{n}|} = \sum_{k=1}^{(K_x-1)\wedge n} v^k =$ __________, $Y_{\overline{x:\overline{n}|}} = \sum_{k=1}^{(K_x-1)\vee n} v^k =$ __________

cts	present value= $\overline{a}_{\overline{n}|} =$ __________	APV
whole life	$\overline{Y}_x = \int_0^{T_x} v^t dt =$ __________	$\overline{a}_x =$ __________dt
n–y. def.	${}_n\|\overline{Y}_x = \int_n^{T_x} v^t dt$__________	${}_n\|\overline{a}_x =$ __________dt
n–y. temp.	$\overline{Y}_{x:\overline{n}\|} = \int_0^{T_x\wedge n} v^t dt =$ __________	$\overline{a}_{x:\overline{n}\|} =$ __________dt
n–y. cer.	$\overline{Y}_{\overline{x:\overline{n}\|}} = \int_0^{n\vee T_x} v^t dt =$ __________	$\overline{a}_{\overline{x:\overline{n}\|}} =$ __________

18. ____ $= \ddot{Y}_{x:\overline{n}|} + {}_n|\ddot{Y}_x$. $E((\ddot{Y}_x)^2)$____$\ddot{a}_x(v^2)$. $\ddot{a}_x =$ __________.

$\overline{a}_x =$ __________. ${}_n|\ddot{a}_x = {}_nE_x\ddot{a}_{x+n} =$ __________.

${}_n|\overline{a}_x = {}_nE_x \cdot \overline{a}_{x+n} =$ __________. $\ddot{a}_{\overline{x:\overline{n+m}|}} = \ddot{a}_{x:\overline{n}|}$__________.

19. Loss in the fully discrete case

Plan	Loss	Loss
Whole life insurance	__________	
t–year funded whole life insurance		__________
n–year term insurance	__________	
t–year funded n–year term insurance		__________
n–year pure endowment insurance	__________	
t–year funded n–year pure endowment insurance		__________
n–year endowment	__________	
t–year funded n–year endowment insurance		__________
n–year deferred insurance	__________	
t–year funded n–year deferred insurance		__________

line-up

Formulas in chapter 7:
1.1. ${}_tL_x = Z_{x+t} - P_x\ddot{Y}_{x+t}$.
1.2. ${}_tV_x = E({}_tL_x)$ $(= A_{x+t} - P_x\ddot{a}_{x+t})$.
1.3. ${}_{t+1}I_x = {}_tV_x + P_x$.
1.4. $P_x\ddot{a}_{x:\bar{t}|} = A^1_{x:\bar{t}|} + {}_tE_x \cdot {}_tV_x$.
1.5. $F = 1 - \frac{P_x}{P_{x+t}}$.
1.6. ${}_tV_x = P_x\frac{\ddot{a}_{x:\bar{t}|}}{{}_tE_x} - \frac{A^1_{x:\bar{t}|}}{{}_tE_x} = P_x\ddot{s}_{x:\bar{t}|} - {}_tk_x$ (whole life insurance).
1.7. ${}_tV + \pi_t = vb_{t+1}q_{x+t} + v \cdot {}_{t+1}Vp_{x+t}$. (general ${}_tV_x$)
1.8. $C_j = vb_{j+1}I(K_x = j+1) - \pi_j I(K_x > j)$.
1.9. $\Lambda_j = C_j + v \cdot {}_{j+1}VI(K_x > j+1) - {}_jVI(K_x > j)$.
1.10. ${}_tL = b_{K_x}v^{K_{x+t}} - \sum_{k=0}^{K_{x+t}-1}\pi_{t+k}v^k = \sum_{k=1}^{\infty} b_{t+k}v^kI(K_{x+t} = k) - \sum_{k=0}^{\infty}\pi_{t+k}v^kI(K_{x+t} > k)$.
1.11. ${}_tV_e = (b+s)A_{x+t} - ((1-r)G - e)\ddot{a}_{x+t} + (e_0^* + r_0^*G)\mathbf{1}(t=0)$
1.12. ${}_tV^e = {}_tV_e - {}_tV$.

2. Formulas in chapter 8:

$T_{xy} = T_{x:y} = \min\{T_x, T_y\}$.

$T_{\overline{xy}} = T_{\overline{x:y}} = \max\{T_x, T_y\}$.

${}_tq_{xy} + {}_tq_{\overline{xy}} = {}_tq_x + {}_tq_y$.

$T_{xy}T_{\overline{xy}} = T_xT_y$.

$T_{xy} + T_{\overline{xy}} = T_x + T_y$.

If $T_x \perp T_y$, then ${}_tq_{\overline{xy}} = {}_tq_x{}_tq_y$, ${}_tp_{xy} = {}_tp_x{}_tp_y$ and $\mu_{xy}(t) = \mu_x(t) + \mu_y(t)$.
3. Shock model: T_x^*, T_y^*, S are independent. $S \sim Exp(\lambda)$. $T_x = T_x^* \wedge S$. $T_y = T_y^* \wedge S$.
$S_{(T_x,T_y)}(s,t) = {}_sp_x^* \cdot {}_tp_y^* e^{-\lambda(s\vee t)}$. ${}_tp_{\overline{xy}} = ({}_tp_x^* + {}_tp_y^* - {}_tp_x^*{}_tp_y^*)e^{-\lambda t}$.

4. Contingent insurance. $\overline{Z}^1_{xy} = v^{T_x}I(T_x < T_y)$. $\overline{Z}_{xy}^{\;\;1} = v^{T_y}I(T_y < T_x)$.
$\overline{Z}^2_{xy} = v^{T_x}I(T_x > T_y)$. $\overline{Z}_{xy}^{\;\;2} = v^{T_y}I(T_y > T_x)$.
5. ${}_nq^1_{xy} = \mathbb{P}\{T_x < T_y, T_x \le n\}$. ${}_nq_{xy}^{\;\;1} = \mathbb{P}\{T_y < T_x, T_y \le n\}$.
${}_nq^2_{xy} = \mathbb{P}\{T_x > T_y, T_x \le n\}$. ${}_nq_{xy}^{\;\;2} = \mathbb{P}\{T_y > T_x, T_y \le n\}$.
6. Revisionary annuity. $\overline{Y}_{x|y} = \int_0^\infty v^tI(T_x \le t \le T_y)dt$. $\overline{a}_{x|y} = \overline{a}_y - \overline{a}_{xy}$.

7. The multiple-decrement model:
1. $T_x^{(1)}, ..., T_x^{(m)}$ are *independent* and cts r.v.
2. $T_x = \min\{T_x^{(1)}, ..., T_x^{(m)}\}$ and $\{J_x = j\} = \{T_x = T_x^{(j)}\}$.
${}_tp_x^{(\tau)} = S_{T_x}(t)$, $\mu_x^{(\tau)}(t) = \mu_{x+t}^{(\tau)} = \frac{f_{T_x}(t)}{S_{T_x}(t)} = \sum_{j=1}^m \mu_x^{(j)}(t)$,
${}_tp_x^{(j)} = P(T_x > t, J_x = j)$. ${}_tp_x^{\prime(j)} = S_{T_x^{(j)}}(t)$.
$f_{(T_x,J_x)}(t,j) = \frac{f_{T_x^{(j)}}(t)}{S_{T_x^{(j)}}(t)}S_{T_x}(t) = \mu_x^{(j)}(t)S_{T_x}(t)$,
$\mu_x^{(j)}(t) = \frac{f_{(T_x,J_x)}(t,j)}{S_{T_x}(t)} = \frac{f_{T_x^{(j)}}(t)}{S_{T_x^{(j)}}(t)} = -\frac{d}{dt}\ln S_{T_x^{(j)}}(t)$.
$G\ddot{a}_x^{(\tau)} = \sum_{k=1}^\infty b_kv^k{}_{k-1}p_x^{(\tau)}q_{x+k-1}^{(1)} + \sum_{k=1}^\infty {}_kCVv^k{}_{k-1}p_x^{(\tau)}q_{x+k-1}^{(2)} + \sum_{k=0}^\infty (r_kG + e_k)v^k{}_kp_x^{(\tau)}$
$({}_kAS + G - Gr_k - e_k)(1+i) = b_{k+1}q_{x+k}^{(1)} + {}_{k+1}CVq_{x+k}^{(2)} + p_{x+k}^{(\tau)} \cdot {}_{k+1}AS$.

Formulas in Pension.

19. $\underline{S_y} = \int_y^{y+1} A_t dt$, $\overline{s}_y/\overline{s}_x = \underline{A_y/A_x}$, $\overline{s}_{x_o} = \underline{1}$, and $\frac{s_y}{s_x} = \frac{\int_0^1 \overline{s}_{y+t}dt}{\int_0^1 \overline{s}_{x+t}dt} = \frac{S_y}{S_x}$.

20. $\underline{\text{The pension replacement ratio } R} = \frac{\text{pension income in the year after retirement}}{\text{final salary before retirement}}$,

21. $B_r = \underline{nS_{Fin}\alpha}$ or nB.

22. ${}_tV = \underline{(t-e)\alpha\hat{S}_{Fin}} \times \ddot{a}_r^{(12)} \times {}_{r-t}p_t v^{r-t}$ if $P(R=r)=1$.

${}_tV = E((t-e)\alpha S_{Fin}\underline{v^{R-t}\ddot{a}_R^{(12)}})$. ${}_tV = \underline{B_t}\frac{D_r^{(r)}}{D_t^{(r)}}\ddot{a}_r^{(12)}$.

23. The normal cost C_t = EPV of benefits for mid-year exists $+\underline{v \cdot {}_1p_x^{00}} \cdot {}_{t+1}V - {}_tV$.

From (8) through (11), assume $\{X_n : n \ge 0\}$ is a homogeneous MC with state space E, one-step transition matrix $P = \big(p_{ij}\big)_{i,j\in E}$ and $p_{ij} = P(i,j)$. Denote m, n, k are nonnegative integers, $q = 1-p$, $\alpha_n(i) = P\{X_n = i\}$.

8. $P\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, ..., X_0 = i_o\} =$ ____________ (key: $P\{X_{n+1} = j|X_n = i\}$)

$\forall\ (n,k)$, $P\{X_{n+k} = j | X_n = i\} = \underline{Q_n^{(k)}(i,j)} =$ ____________ (key: $p_{ij}^{(k)}$)

$P\{X_o = i, X_1 = j, X_2 = k\} = \underline{\alpha_0(i)Q_0(i,j)Q_1(j,k)} =$ ____________ (key: $\alpha_0(i)p_{ij}p_{jk}$)

$P\{X_n = j\} =$ ____________ $=$ ____________ (key: $\sum_{i\in E}\alpha_0(i)p_{ij}^{(n)}$, $\sum_{i\in E}\alpha_1(i)p_{ij}^{(n-1)}$)

$P^{(m)} = (p_{ij}^{(m)})_{i,j\in E}$, $P^{(3)} =$ _____ (key: PPP).

9. Random walk: $E = \{0, \pm1, \pm2, ...\}$. $S_n =$ ____________ (key: $(X_n - X_0 + n)/2) \sim bin(n,p)$.

$(X_{m+n} - X_n)$___ (key: $\perp$) X_n, $(X_{m+n} - X_n)$___ (key: $\sim$) X_m,

$Cov(X_{m+n}, X_n) =$ ___ (key: $V(X_n)) =$ ___ (key: $4npq$).

10. Given $A \subset E$, let $h_i = P(X_n \in A$ for an $n \ge 0 | X_0 = i)$,

h_i's are the minimal nonnegative solutions to the system of equations

$$\begin{cases} h_i = \underline{\qquad\qquad} \text{(key: 1)} & \text{if } i \in A \\ h_i = \underline{\qquad\qquad} \text{(key: } \sum_{j\in E} p_{ij}h_j) & \text{if } i \notin A. \end{cases} \quad (\mathbf{h} = \underline{P}\mathbf{h})$$

11. Gambler's ruin problem. If $E = \{0, 1, ..., N\}$, $p_{i,i-1} = q$ and $p_{i,i+1} = p$ for $i = 1, ..., N-1$, and $p_{00} = 1 = p_{NN}$, then

$$h_i = \begin{cases} \underline{\qquad\qquad} \text{(key: } \frac{1-(q/p)^i}{1-(q/p)^N}) & \text{if } p \ne 1/2 \\ \underline{\qquad\qquad} \text{(key: } i/N) & \text{if } p = 1/2 \end{cases} \text{ with } A = \underline{\quad} \text{ (key: } \{N\})$$

12. Suppose that X_i's are independent Exp(θ_i) r.v.'s.

a. $X_{(1)} \sim$ ____________key: Exp$(\frac{1}{\sum_i 1/\theta_i})$.

b. P$\{X_1 < X_2\} =$ ____________key: $\frac{1/\theta_1}{1/\theta_1 + 1/\theta_2}$.

c. P$\{X_i = X_{(1)}\} =$ ____________key: $\frac{1/\theta_i}{1/\theta_1 + \cdots + 1/\theta_n}$.

13. Let $\{N(t) : t \geq 0\}$ and $\{N_i(t) : t \geq 0\}$ be Poisson processes with rate λ and λ_i, PP(λ) and PP(λ_i). Let $S_n = \inf\{t \geq 0 : N(t) = n\}$ and $T_n = S_n - S_{n-1}$. Let $s, t \geq 0$.
13.1. $N(0) =$ ____________key: 0;
13.2. It has ____________key: independent increments;
13.3. $N(s+t) - N(s) \sim$ ____________key: $Poisson(\lambda t)$.
13.4. Cov(N(s),N(s+t))= ____________key: Var(N(s)).
13.5. T_i's are i.i.d. ____________key: Exp($1/\lambda$).
13.6. $S_n \sim$ ____________key: $\mathcal{G}(n, 1/\lambda)$.
13.7. $N(s+t)|(N(s)) = j) \sim$ ____________key: $N(t) + j$.
13.8. $N(s)|(N(s+t) = n) \sim$ ____________key: $bin(n, p)$, where $p = \frac{s}{s+t}$.
13.9. $N(s)|(S_n = t+s) \sim$ ____________key: $bin(n-1, p)$, where $p = \frac{s}{s+t}$.
13.10. If $\{N_i(t) : t \geq 0\}$'s are independent, then $N_1(t) + N_2(t)$ are also PP($\lambda_1 + \lambda_2$), and $N_1(t)|(N_1(t) + N_2(t) = n) \sim$ ____________key: $bin(n, \frac{\lambda_1}{\lambda_1+\lambda_2})$.
13.11. $\{N(t) < n\} =$ ____________key: $\{S_n > t\}$.

Ignore the rest:
14. Aggregate Claim Model: $S = \sum_{i=1}^{N} Y_i$, where Y_i's are i.i.d. from Y.

$E(S) =$ ____________ (key: $E(N)E(Y)$)

$V(S) =$ ____________ (key: $E(N)V(Y) + V(N)(E(Y))^2$)

15. Nonhomogeneous $PP(m(t))$: $N(t) \sim Pois(m(t))$ with $\underline{m(t)} = \int_0^t \lambda(s)\,ds$. If each time an event occurs it is classified as of the types $1, 2$ with probability $p_j(t)$. Then $N_j(t) \sim PP(m_j(t))$, where $\lambda_j = \underline{\lambda(t)p_j(t)}$.

16. $X_r \sim NB(r, \beta)$: $f_{X_r}(k) = \underline{\binom{r+k-1}{r-1}p^r(1-p)^k}$, $k \geq 0$, $p = \underline{\frac{1}{1+\beta}}$. $E(X_r) = \underline{r\beta}$ and $V(X_r) = \underline{r\beta(\beta+1)}$.

17. If $S_n \sim bin(n, \frac{1}{\beta+1})$, $\mathbb{P}\{X_r \geq k\} = \mathbb{P}\{\underline{S_{r+k-1} \leq r-1}\}$.

18. If $X|(Y = y) \sim Pois(\underline{cy})$ and $Y \sim \mathcal{G}(n, \beta)$, then $X \sim \underline{NB(n, c\beta)}$.

$S_y = \int_y^{y+1} A_t dt$, $\bar{s}_y/\bar{s}_x = A_y/A_x$ and $\frac{s_y}{s_x} = \frac{\int_0^1 \bar{s}_{y+t}dt}{\int_0^1 \bar{s}_{x+t}dt}$, where A_y is the annual rate of salary at age y, S_y is the salary received in year of age y to $y+1$, $\bar{s}_y$ is the rate of salary function, and s_y is the salary scale.

The pension replacement ratio $R = \dfrac{\text{pension income in the year after retirement}}{\text{final salary before retirement}}$,

$B_r = nS_{Fin}\alpha$ or nB.

${}_tV = (t-e)\alpha\hat{S}_{Fin} \times \ddot{a}_r^{(12)} \times {}_{r-t}p_t v^{r-t}$

The normal cost $C_t =$ EPV of benefits for mid-year exists $+v \cdot {}_1p_x^{00} \cdot {}_{t+1}V - {}_tV$.

Formulas for 452.

1. ${}_tL_x =$ ________________. ${}_tV_x = E($_______$)$ $(= A_{x+t} - P_x\ddot{a}_{x+t})$. ${}_{t+1}I_x = {}_tV_x +$ _______.

_______ $= A^1_{x:\overline{t}|} + {}_tE_x \cdot {}_tV_x$. _______ $= 1 - \frac{P_x}{P_{x+t}}$. ${}_tV_x = P_x\frac{\ddot{a}_{x:\overline{t}|}}{{}_tE_x} - \frac{A^1_{x:\overline{t}|}}{{}_tE_x} = P_x\ddot{s}_{x:\overline{t}|} -$ _______.

_______ $+ \pi_t = vb_{t+1}q_{x+t} + v \cdot$ _______p_{x+t}. _______ $= vb_{j+1}I(K_x = j+1) - \pi_j I(K_x > j)$.

_______ $= C_j + v \cdot {}_{j+1}VI(K_x > j+1) - {}_jVI(K_x > j)$.

${}_tL =$ ____________ $- \sum_{k=0}^{K_{x+t}-1} \pi_{t+k}v^k = \sum_{k=1}^{\infty} b_{t+k}v^k I(K_{x+t} = k) - \sum_{k=0}^{\infty} \pi_{t+k}v^k I(K_{x+t} > k)$.

${}_tV_e = (b+s)A_{x+t} -$ ______________$\ddot{a}_{x+t} + (e_0^* + r_0^*G)\mathbf{1}(t=0)$. ________ $= {}_tV_e - {}_tV$.

2. $T_{xy} = T_{x:y} =$ _____$\{T_x, T_y\}$. $T_{\overline{xy}} = T_{\overline{x:y}} =$ _____$\{T_x, T_y\}$. ${}_tq_{xy} + {}_tq_{\overline{xy}} =$ ___________.

____________ $= T_xT_y$. $T_{xy} + T_{\overline{xy}} =$ ___________.

If ____________, then ___________ $= {}_tq_{xt}q_y$, ___________ $= {}_tp_{xt}p_y$ and ___________ $= \mu_x(t) + \mu_y(t)$.

3. Shock model: ____________ are independent. ____________. $T_x = T_x^* \wedge S$. ____________.

____________ $= {}_sp_x^* \cdot {}_tp_y^* e^{-\lambda(s \vee t)}$. ________ $= ({}_tp_x^* + {}_tp_y^* - {}_tp_{xt}^*p_y^*)e^{-\lambda t}$.

4. Contingent insurance. $\overline{Z}^1_{xy} =$ ______________. $\overline{Z}_{x\overset{1}{y}} =$ ______________.

______________ $= v^{T_x}I(T_x > T_y)$. ______________ $= v^{T_y}I(T_y > T_x)$.

5. ${}_nq^1_{xy} = \mathbb{P}\{$______________$, T_x \le n\}$. ${}_nq_{x\overset{1}{y}} = \mathbb{P}\{$______________$, T_y \le n\}$.

${}_nq^2_{xy} = \mathbb{P}\{$______________$, T_x \le n\}$. ${}_nq_{x\overset{2}{y}} = \mathbb{P}\{$______________$, T_y \le n\}$.

6. Revisionary annuity. $\overline{Y}_{x|y} = \int_0^\infty$ ______________dt. $\overline{s}_{x|y} = \overline{a}_y -$ ______________.

7. The multiple-decrement model:

$T_x^{(1)}, ..., T_x^{(m)}$ are ___________ and cts r.v. _____ $= \min\{T_x^{(1)}, ..., T_x^{(m)}\}$ and ________ $= \{T_x = T_x^{(j)}\}$.

$\underline{\hspace{2cm}} = S_{T_x}(t)$, $\mu_x^{(\tau)}(t) = \mu_{x+t}^{(\tau)} = \frac{f_{T_x}(t)}{S_{T_x}(t)} = \underline{\hspace{3cm}}$, $\underline{\hspace{2cm}} = P(T_x > t, J_x = j)$. $\underline{\hspace{2cm}} = S_{T_x^{(j)}}(t)$.

$\underline{\hspace{2cm}} = \frac{f_{T_x^{(j)}}(t)}{S_{T_x^{(j)}}(t)} S_{T_x}(t) = \mu_x^{(j)}(t) S_{T_x}(t)$, $\underline{\hspace{2cm}} = \frac{f_{(T_x,J_x)}(t,j)}{S_{T_x}(t)} = \frac{f_{T_x^{(j)}}(t)}{S_{T_x^{(j)}}(t)} = -\frac{d}{dt}\ln S_{T_x^{(j)}}(t)$.

$G\ddot{a}_x^{(\tau)} = \sum_{k=1}^{\infty} \underline{\hspace{2cm}} v^k{}_{k-1}p_x^{(\tau)} q_{x+k-1}^{(1)} + \sum_{k=1}^{\infty} \underline{\hspace{2cm}} v^k{}_{k-1}p_x^{(\tau)} q_{x+k-1}^{(2)} + \sum_{k=0}^{\infty} \underline{\hspace{3cm}} v^k{}_k p_x^{(\tau)}$

$(\underline{\hspace{2cm}} + G - Gr_k - e_k)(1+i) = b_{k+1} q_{x+k}^{(1)} + {}_{k+1}CV q_{x+k}^{(2)} + p_{x+k}^{(\tau)} \cdot \underline{\hspace{2cm}}$.

19. $\underline{\hspace{1cm}} = \int_y^{y+1} A_t dt$, $\overline{s}_y/\overline{s}_x = \underline{A_y/A_x}$, $\overline{s}_{x_o} = \underline{\hspace{1cm}}$, and $\frac{s_y}{s_x} = \int_0^1 \underline{\hspace{2cm}} dt / \int_0^1 \underline{\hspace{2cm}} dt = \frac{S_y}{S_x}$.

20. $\underline{\hspace{8cm}} = \frac{\text{pension income in the year after retirement}}{\text{final salary before retirement}}$,

21. $B_r = \underline{\hspace{3cm}}$ or nB.

22. ${}_tV = \underline{\hspace{3cm}} \times \ddot{a}_r^{(12)} \times {}_{r-t}p_t v^{r-t}$ if $P(R = r) = 1$.

${}_tV = E((t-e)\alpha S_{Fin} \underline{\hspace{3cm}})$. ${}_tV = \underline{\hspace{2cm}} \frac{D_r^{(r)}}{D_t^{(r)}} \ddot{a}_r^{(12)}$.

23. The normal cost $C_t =$ EPV of benefits for mid-year exists $+ \underline{\hspace{3cm}} \cdot {}_{t+1}V - {}_tV$.

From (8) through (12), assume $\{X_n : n \geq 0\}$ is a homogeneous MC with state space E, one-step transition matrix $P = \left(p_{ij}\right)_{i,j \in E}$ and $p_{ij} = P(i,j)$. Denote m, n, k are nonnegative integers, $q = 1 - p$, $\alpha_n(i) = P\{X_n = i\}$.

8. $P\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, ..., X_0 = i_o\} = \underline{\hspace{3cm}}$.

$\forall\ (n,k)$, $P\{X_{n+k} = j | X_n = i\} = \underline{\hspace{3cm}} = \underline{\hspace{3cm}}$.

$P\{X_o = i, X_1 = j, X_2 = k\} = \underline{\hspace{5cm}} = \underline{\hspace{3cm}}$.

$P\{X_n = j\} = \underline{\hspace{3cm}} = \underline{\hspace{3cm}}$. $P^{(m)} = (p_{ij}^{(m)})_{i,j \in E}$, $P^{(3)} = \underline{\hspace{1cm}}$.

9. Random walk: $E = \{0, \pm 1, \pm 2, ...\}$. $S_n = \underline{\hspace{3cm}} \sim bin(n,p)$.

$(X_{m+n} - X_n) \underline{\hspace{0.8cm}} X_n$, $(X_{m+n} - X_n) \underline{\hspace{0.8cm}} X_m$, $Cov(X_{m+n}, X_n) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

10. Given $A \subset E$, let $h_i = P(X_n \in A \text{ for an } n \geq 0 | X_0 = i)$, h_i's are

the minimal nonnegative solutions to the system of equations. $h_i = \begin{cases} \_\_\_\_\_\_\_\_\_\_ & \text{if } i \in A \\ \_\_\_\_\_\_\_\_\_\_ & \text{if } i \notin A. \end{cases}$

($\mathbf{h} =$ ___$\mathbf{h}$).

12. Suppose that X_i's are independent Exp(θ_i) r.v.'s.

a. $X_{(1)} \sim$ ____________. b. $P\{X_1 < X_2\} =$ ____________. c. $P\{X_i = X_{(1)}) =$ ____________.

13. Let $\{N(t) : t \geq 0\}$ and $\{N_i(t) : t \geq 0\}$ be Poisson processes with rate λ and λ_i, PP(λ) and PP(λ_i).

Let $S_n = \inf\{t \geq 0 : N(t) = n\}$ and $T_n = S_n - S_{n-1}$. Let $s, t \geq 0$.

13.1. $N(0) =$ ___. 13.2. It has ______________________. 13.3. $N(s+t) - N(s) \sim$ ____________.

13.4. Cov(N(s),N(s+t))= ____________. 13.5. T_i's are i.i.d. ____________. 13.6. $S_n \sim$ ____________.

13.7. $N(s+t)|(N(s)) = j) \sim$ ____________. 13.8. $N(s)|(N(s+t) = n) \sim$ ____________.

13.9. $N(s)|(S_n = t+s) \sim$ ______________________. 13.10. If $\{N_i(t) : t \geq 0\}$'s are independent, then $N_1(t) + N_2(t)$

are also PP($\lambda_1 + \lambda_2$), and $N_1(t)|(N_1(t) + N_2(t) = n) \sim$ ____________. 13.11. $\{N(t) < n\} =$ ____________.

1. Axioms of probability: (1) $P(A) \geq$ ____, (2) $P(S) =$ ___, ...

2. $P(A \cap B) =$ ________. If A and B are ________________ then $P(A \cap B) = P(A)P(B)$.

3. $P($_____$)=1 - P(A)$. $P(A \cup B) = P(A) + P(B)$ ________________.

   If A and B are ______________ then $P(A \cup B) = P(A) + P(B)$.

4. $X \sim bin(n, p)$: $f(i) =$ ________________ if $i \in \{0, ..., n\}$, $\mu =$ ______, $\sigma^2 =$ ______, where $q = 1 - p$

5. $X \sim Pois(\lambda)$. $f(i) =$ ________________ if $i \geq 0$. $\mu =$ ________, $\sigma^2 =$ ________

6. $Y = g(X)$. $E(g(X)) = \begin{cases} \sum_y y f_Y(y) & dis \\ \int y f_Y(y)dy & cts \end{cases} = \begin{cases} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ & dis \\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ & cts \end{cases}$, $\mu_Y =$ _____, $\sigma^2_Y =$ _____

7. The mgf of X is $M(t) =$ ___________, $\frac{d^k M(t)}{dt^k}\Big|_{t=0} =$ ________________

8. A cdf $F(t)$ (= ___________) satisfying (1) $F(-\infty) =$ _____, and $F(\infty) =$ _____, (2) $F(x+) =$ _____,

   (3) $F(x)$ _____ . Moreover, $F(b) - F(a) = P($__________$)$

9. $F(t) = \begin{cases} \sum_{x \leq t} \_\_\_\_\_\_\_\_ & dis \\ \int_{-\infty}^t \_\_\_\_\_\_\_\_\_\_\_\_ & cts \end{cases}$, $f(t) = \begin{cases} \_\_\_\_\_\_\_\_\_\_\_\_ & dis \\ \_\_\_\_\_\_\_\_ & cts \end{cases}$

10. $E(aX + b) =$ ________________. $Var(aX + b) =$ ________________

11. $X \sim N(\mu, \sigma^2)$. $f(x) =$ ______________________, $\frac{X-\mu}{\sigma} \sim$ ________________

12. $X \sim \mathcal{G}(\alpha, \beta)$. $f(x) =$ ________________ if $x >$ _, $\mu =$ ______, $\sigma^2 =$ ______, $\Gamma(\alpha + 1) =$ ________

13. $Exp(\lambda) =$ ________________, $\chi^2(\nu) =$ ________________,

14. $f_X(x) = \begin{cases} \int f(x, y)\_\_\_\_\_\_\_ & cts \\ \_\_\_\_\_\_\_ f(x, y) & dis \end{cases}$ 15. $E(g(X, Y)) = \begin{cases} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ & cts \\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ & dis \end{cases}$

16. $f_{X|Y}(x|y) =$ ____________, $F_{X|Y}(x|y) = P$ (____________)

17. $E(c) =$ ____, $E(ag(X,Y) + bh(X,Y)) =$ ________________,

18. $Cov(X,Y) =$ ______________, $V(aX + bY) =$ ________________________, $\rho(X,Y) =$ _____

19. $E(X|Y = y) = \begin{cases} \_\_\_\_\_\_\_\_\_\_\_\_ & cts \\ \_\_\_\_\_\_\_\_\_\_\_\_ & dis \end{cases}$. $E(E(X|Y)) =$ ______, $E(V(X|Y)) + V(E(X|Y)) =$ ______.

20. $U = h(Y)$, where h is ____________ and Y is cts. $f_U(u) = f_Y($______$) \cdot$ ______

21. Let Y_1, ..., Y_n be a random sample of Y. $\overline{Y} =$ __________________, $S^2 = S_Y^2 =$ ____________, $\mu_{\overline{Y}} =$ ____________, and $\sigma^2_{\overline{Y}} =$ ____________.

22. $F_{\overline{Y}}(t)$ ____$\Phi($________$)$, where $\Phi(t)$ is the cdf of _______.

44. If X_1___X_2.

X_i's $\sim$:	$X_1 + X_2 \sim$:
$G(\alpha_i, \beta)$	__________
$\chi^2(v_i)$	__________
$Pois(\lambda_i)$	__________
$N(\mu_i, \sigma_i^2)$	__________
$bin(n_i, p)$	__________

Answer Keys can be found from:
http://people.math.binghamton.edu/qyu/ftp/form2.pdf