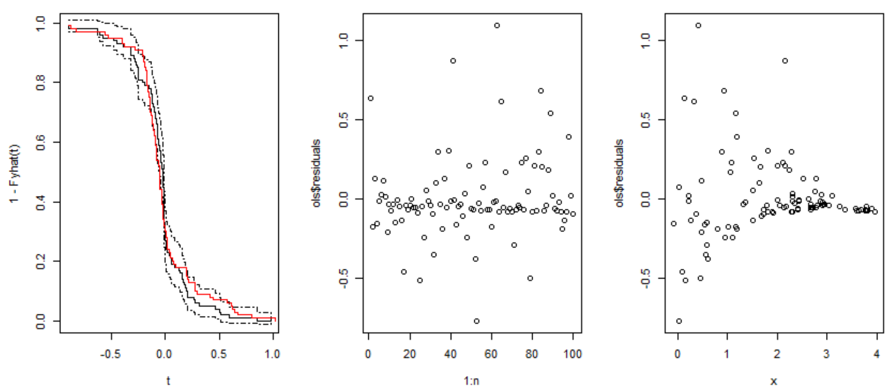
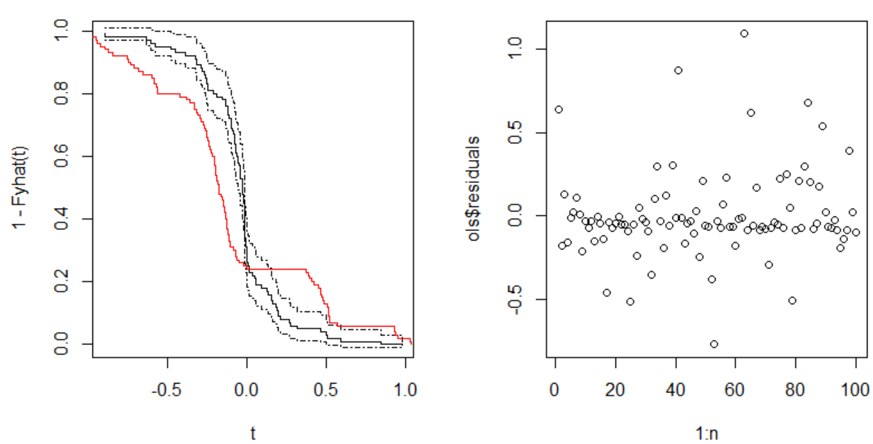
556 Homework 6

Name: Xiaoke Qin

1. (a) Under the Cox model, if we use naïve method, we have the following figures:



Under the Cox model, if we bootstrap the W^* when $|X|$ is around 0, we have the following figures:



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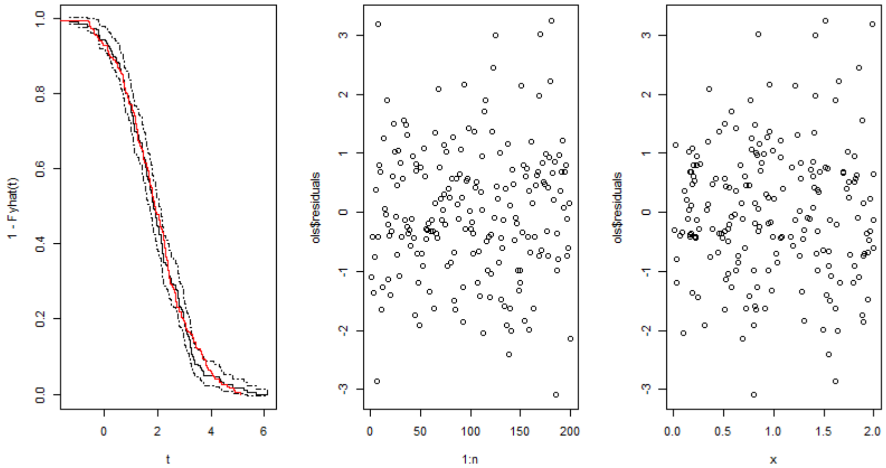

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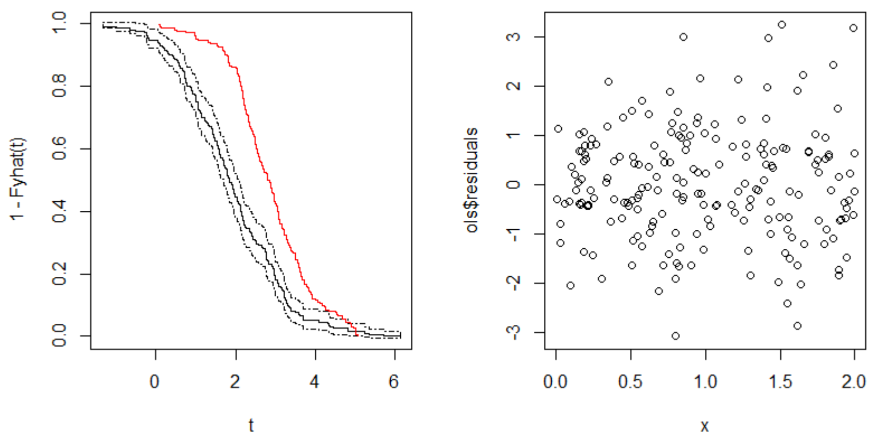


(b)
```
t1<-function(x,y){
a=Mode(x)
x=x-a
ols<-lm(y~x)
c=abs(max(x)-min(x))/2
deltan<-c*n^(-1/(3))
Fy<-ecdf(y)
t<-knots(Fy)
Fyex<-c(0,Fy(t))
fy<-Fyex[2:length(Fyex)]-Fyex[1:(length(Fyex)-1)]

indexM<-which(abs(x)<=deltan)
```

```
X0<-x[indexM]
m<-length(X0)
Xnon0<-x[-indexM]

newW<-sample(y[indexM],m,replace = T)
newY<-rep(predict(ols,newdata = as.data.frame(x)),each=m)+rep(newW,n)
Fystar<-ecdf(newY)

T1<-sum(abs(Fy(t)-Fystar(t))*fy)
return(T1)
}

Mode<-function(sta){
h<-density(sta)
a<-h$x[which.max(h$y)]
a
}

n=100
k=n**0.7
N=100
rn=100
set.seed(1027)
count=0

for(k in 1:N){
x=runif(n,-4/k,4)
y=rexp(n,exp(x))-exp(-x)
a<-Mode(x)
x=x-a
ols<-lm(y~x)
c=abs(max(x)-min(x))/2
deltan<-c*n^(-1/(3))
X0<-x[abs(x)<=deltan]
m<-length(X0)
Xnon0<-x[abs(x)>deltan]

T0<-t1(x,y)
T1j<-rep(0,rn)
for (i in 1:rn){
  newX<-c(sample(X0,m,replace=T),sample(Xnon0,n-m,replace=T))
  newX<-sample(newX)
  newW<-sample(y[abs(x)<=deltan],n,replace = T)
  newY<-predict(ols,newdata = as.data.frame(newX))+newW
  T1j[i]<-t1(newX,newY)
}
Ft<-ecdf(T1j)
if (Ft(T0)<0.95){
  count=count+1
}
}
```

We have the table

	n	T_1
Cox	60	0.08
	200	0.01
	300	0.00
WLR	60	0.00
	120	0.00

2. Assuming we have model:

$$Y_{ij} = \mu + \tau_j + \varepsilon_{ij}, \varepsilon_{ij} \sim N(0, \sigma^2) \quad i.i.d.$$

$j = 1, 2, 3$ means the corresponding observation is from team A,B,C respectively.
By OLS and anova, we have

```
Analysis of Variance Table

Response: x
          Df Sum Sq Mean Sq F value   Pr(>F)
team       2     98  49.000       9 0.001953 **
Residuals 18     98   5.444
---

```

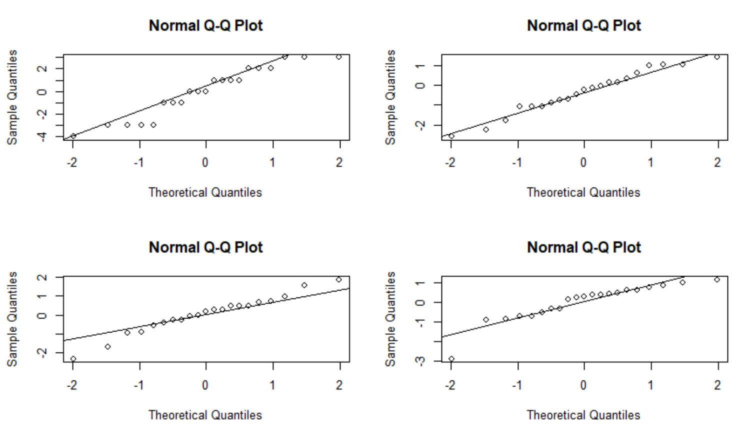
Suppose we have hypothesis

$$H_0 : \tau_1 = \tau_2 = \tau_3$$

and alternative hypothesis

$$H_1 : \text{the equation is not true for at least one pair of } \tau_i, \tau_j.$$

According to the result of anova, P-value is smaller than 0.05, so we reject null hypothesis and conclude that team A, B, C are significantly different. Normality checking:



3. (a) We have $x_1 + \hat{\alpha}_1 + \hat{\gamma}_1 = \bar{Y} + \bar{Y}_{1\cdot} - \bar{Y} + \bar{Y}_{\cdot 1} - \bar{Y} = \bar{Y}_{1\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$.
Since $\hat{\alpha}_1 = \hat{\gamma}_1 = 0$, we have $x_1 = \bar{Y}_{1\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$.

   (b) $x_1 + x_2 = \tilde{\eta} + \tilde{\gamma}_2 + \tilde{\alpha}_1 = \bar{Y} + (0 - (\bar{Y}_{\cdot 1} - \bar{Y})) + \bar{Y}_{1\cdot} - \bar{Y} = \bar{Y}_{\cdot 2} + \bar{Y}_{1\cdot} - \bar{Y}$.
Since $x_1 = \bar{Y}_{1\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$, $x_2 = \bar{Y}_{\cdot 2} - \bar{Y}_{\cdot 1}$.

   (c) $x_1 + x_3 = \bar{Y} + \bar{Y}_{\cdot 1} - \bar{Y} + \bar{Y}_{2\cdot} - \bar{Y} = \bar{Y}_{2\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$.
Since $x_1 = \bar{Y}_{1\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$, $x_3 = \bar{Y}_{2\cdot} - \bar{Y}_{1\cdot}$.

   (d) $x_1 + x_4 = \bar{Y} + \bar{Y}_{\cdot 1} - \bar{Y} - (\bar{Y}_{1\cdot} - \bar{Y} + \bar{Y}_{2\cdot} - \bar{Y}) = \bar{Y}_{\cdot 1} + \bar{Y}_{3\cdot} - \bar{Y}$
Since $x_1 = \bar{Y}_{1\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$, $x_4 = \bar{Y}_{3\cdot} - \bar{Y}_{1\cdot}$

   (e) $x_5 = x_1 = \bar{Y}_{1\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$.

   (f) $x_6 = x_1 + x_2 = \bar{Y}_{\cdot 2} + \bar{Y}_{1\cdot} - \bar{Y}$.

   (g) $x_5 + x_7 = x_1 + x_3 = \bar{Y}_{2\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$.
Since $x_5 = \bar{Y}_{1\cdot} + \bar{Y}_{\cdot 1} - \bar{Y}$, $x_7 = \bar{Y}_{2\cdot} - \bar{Y}_{1\cdot}$.

   (h) $x_8 + x_5 = x_1 + x_4$. Since $x_1 = x_5$, $x_8 = x_4 = \bar{Y}_{3\cdot} - \bar{Y}_{1\cdot}$

4. $$\mathbf{Y} = \begin{pmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \\ Y_{31} \\ Y_{32} \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\beta = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$