Remark 7. Example 4 illustrates that one should simplify the model as much as possible in data analysis. **How ?**
The data in Example 4 satisfy the Weibull distribution:
$\ln Y = \beta_o + \beta_1 x + \tau \ln Z$, with $Z \sim Exp(1)$, $(\beta_0, \beta_1, \tau) = (0.1, 1.95, e^{0.07})$? Or $(0, 2, 1)$?
The data fit Weibull() better than Exponential() (in CB), but one needs to check whether the data are indeed from Weibull **(how ?)** and try to simplify it.
The more the parameter, the less the efficiency, thus the SD increases if # of parameters increases (it is why $Log(scale)$ is significant with p-value 0.02×2).

	weibull	*gamma or exp*
β_o	0.12	0.11
β_1	0.044	0.042

The simplified model is $\ln Y = \beta_o + \beta_1 x + \tau \ln Z$, with $(\beta_0, \beta_1, \tau) = (0, 2, 1)$ and $Z \sim Exp(1)$, and becomes $Exp(\mu)$ with mean e^{2x}). **How to test it ?**

Example 5. (Simulation study). Generate data from $Y \sim Exp(\mu)$, where

$S(t) = \exp(-\lambda t) = \exp(-\frac{t}{\mu})$, $t > 0$,

$Z = \frac{Y}{\mu} \sim Exp(1)$.
$\ln Y = \ln\mu + \ln Z$.
> library(MASS) ; library(survival) ; n=500 ; y=rexp(n,rate=5);
Then pretend we do not know it. Estimate $E(Y)$ by the MLE based on Weibull, Exp or Gamma and by the LSE, using varies codes:
non-regression:

fitdistr(y,"exponential"), fitdistr(y,"weibull"), fitdistr(y,"gamma")

regression and parametric:

survreg(Surv(y)~ 1),
survreg(Surv(y)~ 1, dist="exponential")
glm(y~1,family=Gamma(link=log))

semi-parametric:

lm(log(y)~1)
lm(y~1)

Question: What is the true value that is estimated ?
How many distinct values of the estimates ?

> (z=fitdistr(y,"exponential")$e)
rate $\quad \lambda = 1/scale\ S(t) = \exp(-\lambda t)$
5.235007 $\quad \hat\lambda = 1/\hat\mu$
> 1/z
0.1910217 $\quad \hat\mu$
> (z=survreg(Surv(y)~1, dist="exponential")$co) $\quad Y = e^{\alpha} Z$
-1.655368 $\quad \hat\alpha$
> exp(z)
0.1910217 $\quad \hat\mu = e^{\hat\alpha}$
> predict(z,data.frame(x=0))
0.1910217 $\quad \hat\mu$
> z=glm(y~1,family=Gamma(link=log),maxit=50)
> z[[1]] $\quad$ (= link $\ln\mu = \alpha$)