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Notations in 450: \( k, i, n \geq 0 \) and \( 0 \leq m \leq n \). \( P(X \geq 0) = 1 \) and \( x, t > 0 \). \( S_X(x) = s(x) = P(X > x) \).

1. If \( H(0) = 0 \) and \( H' \geq 0 \), then \( E(H(X)) = \int_0^\infty s(t)H'(t) \, dt \), e.g., \( E(X) = \int_0^\infty s(t) \, dt \), \( E(X^p) = \int_0^\infty s(t)t^{p-1} \, dt \), \( E[X \wedge a] = \int_0^a s(t) \, dt \).

2. If \( P(X \in \{0, 1, 2, \ldots\}) = 1 \) and \( H \uparrow \), then \( E[H(X)] = \sum_{k=1}^{\infty} \mathbb{P}\{X \geq k\}(H(k) - H(k-1)) \), \( E[X] = \sum_{k=1}^{\infty} \mathbb{P}\{X \geq k\} \).

3. \( T(x) = T_x = (X - x) X > x \),
\[ tP_x = S_{T(x)}(t) = \frac{s(x+t)}{s(x)} \cdot t\varphi_x = F_{T(x)}(t) = \frac{s(x)-s(x+t)}{s(x)} \cdot \varphi_x, \]
\[ s\varphi_x = \mathbb{P}\{s < T(x) \leq s + t\} = \varphi_x \cdot |q_{x+s}|, \]
\[ s\varphi_x = \mathbb{P}\{s \mid T(x) \leq s + t\} = \varphi_x \cdot q_x, \]
\[ m_x = \frac{n_x}{n} \frac{s(x)}{s(x+t)} \cdot \varphi_x, \]
\[ m_x = \varphi_x \cdot g_x, \]
\[ m = \varphi_x \cdot g_x. \]

4. The force of mortality is \( \mu_X = \mu = \frac{f_X(x)}{S_X(x)} \). If \( X \) is cts, \( \mu(x) = -\frac{d}{dx} \ln S_X(x), \frac{S_X(x)}{S_X(x-)} = \exp \left( -\int_0^x \mu(t) \, dt \right) \).

5. The central rate of failure on \((x, x + n)\] is \( n_s = \frac{x^n}{n} \frac{x^n}{s(x+t)} \cdot \varphi_x \).

6. \( K_x = [T(x)], \[ t \] = k \) if \( t \in (k - 1, k], K(x) = K_x \).

7. The Nelson-Aalen estimator: \( \hat{S}_{NA}(t) = e^{-H(t)}1(t > 0) \), where \( H(t) = \sum_{k \leq t} \frac{d_k}{x_k} \).

8. The Kaplan-Meier estimator: \( \hat{S}(t_k) = \prod_{k \leq t} \left( 1 - \frac{d_k}{x_k} \right) \), and \( \hat{S}'(t_k) = \frac{1}{n} \hat{S}'(t_k) \).

9. \( \ell_x = \# \) of individuals alive at age \( x \), \( \ell_x = L_x, \ell_x = s(x), \ell_x = t \).

10. The key to computing expected lives:
\[ UDD \]
\[ \text{exponential} \]
\[ \text{Balducci} \]
\[ \hat{S}_{NA}(t) = \left( \frac{\ell_x + t \ell_x}{\ell_x} \right)^{\frac{1}{\ell_x}} \]
14. Life Insurance:  $Z = b_x v_r t_x$.
Whole life ins:  $Z_x = \frac{v R_x}{a_x}, \quad \overline{Z}_x = \frac{v T_x}{a_x}, \quad A_x = A_x(v) = E[vK_x] = v q_x + v p_x A_{x+1}, \quad 2A_x = E[\overline{Z}_x^2] = A_x(v^2)$.

$n$-year term:  $Z_{x:1\overline{m}} = v K_x I(K_x \leq n), \quad \overline{Z}_{x:1\overline{m}} = v T_x I(T_x \leq n), \quad A_{x:1\overline{m}} = E[\overline{Z}_{x:1\overline{m}}] = \sum_{k=1}^n v^k f_{K_x}(k) = v q_x + v p_x A_{x+1:n-1}$,  $2A_{x:1\overline{m}} = E((\overline{Z}_{x:1\overline{m}})^2) = A_{x:1\overline{m}}(v^2)$.

$n$-year deferred:  $\overline{Z}_x = v K_x I(n < K_x), \quad \overline{Z}_x = v T_x I(n < T_x), \quad 2n A_x = n A_x(v^2), \quad n A_x = E[n|Z_x] = \sum_{k=1}^\infty v^k f_{K_x}(k) = v^{n+1} f_{K_x}(n+1) + n+1 A_x = v p_x E[n|A_{x+1}]

$n$-year pure endowment:  $Z_{x:1\overline{m}} = v^n I(n < K_x), \quad \overline{Z}_{x:1\overline{m}} = v^n I(n < T_x), \quad A_{x:1\overline{m}} = n E_x = \sum_{k=1}^n v^k f_{K_x}(k) = \sum_{k=1}^n v^k f_{K_x}(k) = v^{n+1} f_{K_x}(n+1) + n+1 A_x = v p_x E[n|A_{x+1}]

$m$-year defer $n$-year term:  $m n Z_x = v K_x I(m < K_x \leq n + m), \quad m n \overline{Z}_x = v T_x I(m < T_x \leq n + m), \quad m n A_x = m E_x A_{x+m,1\overline{m}}$,  $2m A_{x:1\overline{m}} = E((m n Z_x)^2) = \sum_{k=1}^{n+m} v^{2k} f_{K_x}(k)$.

\[Z_x = Z_{x:1\overline{m}} + n |Z_x, \quad Z_{x:1\overline{m}} = n |Z_x, \quad Z_{x:1\overline{m}} = Z_{x:1\overline{m}}, \quad Z_{x:1\overline{m}} = Z_{x:1\overline{m}}, \quad Z_{x:1\overline{m}} = Z_{x:1\overline{m}}.
\]

\[\overline{Z}_x = \overline{Z}_{x:1\overline{m}} + n |\overline{Z}_x, \quad \overline{Z}_{x:1\overline{m}} = n |\overline{Z}_x, \quad \overline{Z}_{x:1\overline{m}} = \overline{Z}_{x:1\overline{m}}, \quad \overline{Z}_{x:1\overline{m}} = \overline{Z}_{x:1\overline{m}}, \quad \overline{Z}_{x:1\overline{m}} = \overline{Z}_{x:1\overline{m}}.
\]

15. (IZ)$_x = K_x v Z_x$,  (TZ)$_x = T_x v T_x$,  (IZ)$_x = [T_x] v T_x$  (DZ)$_x = (n+1-K_x) v K_x I(K_x \leq n)$.

\[(DZ)_{x:1\overline{m}} = (n-T_x) v T_x I(T_x \leq n), \quad (DZ)_{x:1\overline{m}} = [n-T_x] v T_x I(T_x \leq n).
\]

16. $v = \frac{1}{i+1}$,  $\delta = -ln v$,  $d = 1 - v$.  $\sum_{k=1}^n t x^{k-1} = \left(\frac{1-x^{n+1}}{1-x}\right)^d$.

\[a_{\overline{m}} = \sum_{k=1}^n v^k = v\left(\frac{1-x^n}{1-v}\right)
\]

<table>
<thead>
<tr>
<th>due</th>
<th>PV</th>
<th>APV</th>
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<tr>
<td>whole life</td>
<td>$Y_x = \sum_{k=0}^{K_x-1} v^k = \frac{1-Z_x}{1-v}$</td>
<td>$\tilde{a}<em>x = \sum</em>{k=0}^{\infty} v^k p_x$</td>
</tr>
<tr>
<td>$n$-y. def.</td>
<td>$n</td>
<td>Y_x = \sum_{k=n}^{K_x-1} v^k = \frac{v^n - v^{K_x}}{1-v} I(K_x &gt; n)$</td>
</tr>
<tr>
<td>$n$-y. tem.</td>
<td>$\tilde{Y}<em>{x:1\overline{m}} = \sum</em>{k=n}^{K_x-1} v^k = \frac{1-Z_{x:1\overline{m}}}{d}$</td>
<td>$n</td>
</tr>
<tr>
<td>$n$-y. cer.</td>
<td>$\tilde{Y}<em>{x:1\overline{m}} = \sum</em>{k=n}^{(K_x-1)\cap n} v^k = \tilde{a}_{\overline{m}} + n</td>
<td>Y_x$</td>
</tr>
</tbody>
</table>

immediate:  $Y_x = \sum_{k=0}^{K_x-1} v^k = \tilde{Y}_x - 1$,  $n |Y_x = \sum_{k=n}^{K_x-1} v^k = n |\tilde{Y}_x$.

$Y_x = \sum_{k=1}^{K_x-1} v^k = \tilde{Y}_{x:1\overline{m}} - 1$,  $Y_{x:1\overline{m}} = \sum_{k=1}^{(K_x-1)\cap n} v^k = \tilde{Y}_{x:1\overline{m}} - 1$.
### 18. \( \bar{Y}_x \) and \( \bar{Y}_x \) as Functions of \( \bar{Y}_x \)

\[
\bar{Y}_x = \bar{Y}_{x:n} + n \bar{Y}_x, \quad E((\bar{Y}_x)^2) + \bar{a}_x(v^2), \quad \bar{a}_x = 1 + vp_x \bar{a}_{x+1}, \quad \bar{a}_x = \bar{a}_{x:n} + v^n p_x \bar{a}_{x+n}.
\]

### 19. Plan and Loss

<table>
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<th>Loss</th>
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<td>Whole life insurance</td>
<td>( Z_x - P\bar{Y}_x )</td>
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<tr>
<td>t-year funded whole life insurance</td>
<td>( Z_x - P\bar{Y}_{x:n} )</td>
</tr>
<tr>
<td>n-year term insurance</td>
<td>( Z_{x:n} - P\bar{Y}_{x:n} )</td>
</tr>
<tr>
<td>t-year funded n-year term insurance</td>
<td>( Z_{x:n} - P\bar{Y}_{x:n} )</td>
</tr>
<tr>
<td>n-year pure endowment insurance</td>
<td>( Z_{x:n}^{-1} - P\bar{Y}_{x:n} )</td>
</tr>
<tr>
<td>t-year funded n-year pure endowment insurance</td>
<td>( Z_{x:n}^{-1} - P\bar{Y}_{x:n} )</td>
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<tr>
<td>n-year endowment</td>
<td>( Z_{x:n} - P\bar{Y}_{x:n} )</td>
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<tr>
<td>t-year funded n-year endowment insurance</td>
<td>( Z_{x:n} - P\bar{Y}_{x:n} )</td>
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<tr>
<td>n-year deferred insurance</td>
<td>( n</td>
</tr>
<tr>
<td>t-year funded n-year deferred insurance</td>
<td>( n</td>
</tr>
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</table>

Loss in the fully discrete case

### 13. Application of Woolhouse’s formula:

\[
\bar{a}_x \approx \bar{a}_x - \frac{1}{2} \cdot \frac{1}{12} (\delta + \mu_x) \approx \bar{a}_x^{(m)} - \frac{1}{2m} - \frac{1}{12m^2}(\delta + \mu_x)
\]
Syllabus for Math 450:
The material will focus on Long-Term Actuarial Mathematics Exam.
(1) Various Life insurance such as whole life, $n$–year term, $n$–year deferred, $n$–year endowment or pure endowment, and $m$–year deferred $n$-year term life insurances.
(2) Various Life annuities such as (due, immediate and continuous), whole life, $n$–year deferred, $n$–year temporary, $n$–year certain annuities with level or non-level payments.
(3) Premiums calculation for insurance and annuity using equivalence principle,
(4) Present value random variable (rv) and the random variables involves in (1), (2) and (3), including loss-at-issue r.v. for premiums. Compute their probability, means, variance, percentiles, force of mortality and central rate of failure with changes in mortality and interest, under select and ultimate survival models, parametric model and tabular model, select and ultimate mortality table, or using approximation methods such as UUD, constant force, Woolhouse and Euler, Kaplan-Meier estimator and Nelson-Aalen estimator.
1. Axioms of probability: (1) \( P(A) \geq 0 \), (2) \( P(S) = 1 \).

2. \( P(A \cap B) = \ldots \). If \( A \) and \( B \) are \ldots then \( P(A \cap B) = P(A)P(B) \).

3. \( P(\ldots) = 1 - P(A) \). \( P(A \cup B) = P(A) + P(B) \ldots \).

   If \( A \) and \( B \) are \ldots then \( P(A \cup B) = P(A) + P(B) \).

4. \( X \sim \text{bin}(n, p) \): \( f(i) = \ldots \) if \( i \in \{0, \ldots, n\} \), \( \mu = \ldots \), \( \sigma^2 = \ldots \), where \( q = 1 - p \)

5. \( X \sim \text{Pois}(\lambda) \). \( f(i) = \ldots \) if \( i \geq 0 \). \( \mu = \ldots \), \( \sigma^2 = \ldots \)

6. \( Y = g(X) \). \( E(g(X)) = \left\{ \begin{array}{ll} \sum_y yf_Y(y) & \text{dis} \\ \int yf_Y(y)dy & \text{cts} \end{array} \right. \ldots \)

7. The mgf of \( X \) is \( M(t) = \ldots \) \( \frac{d^k M(t)}{dt^k} \bigg|_{t=0} = \ldots \)

8. A cdf \( F(t) (= \ldots) \) satisfying (1) \( F(-\infty) = \ldots \), and \( F(\infty) = \ldots \). (2) \( F(x+) = \ldots \)

(3) \( F(x) \ldots \). Moreover, \( F(b) - F(a) = P(\ldots) \)

9. \( F(t) = \left\{ \begin{array}{ll} \sum_{x \leq t} & \text{dis} \\ d(t) & \text{cts} \end{array} \right. \ldots \)

10. \( E(aX + b) = \ldots \), \( Var(aX + b) = \ldots \)

11. \( X \sim N(\mu, \sigma^2) \). \( f(x) = \ldots \), \( \frac{X-\mu}{\sigma} \sim \ldots \)

12. \( X \sim G(\alpha, \beta) \). \( f(x) = \ldots \) if \( x > \ldots \), \( \mu = \ldots \), \( \sigma^2 = \ldots \), \( \Gamma(\alpha + 1) = \ldots \)

13. \( \text{Exp}(\lambda) = \ldots \), \( \chi^2(\nu) = \ldots \)

14. \( f_X(x) = \left\{ \begin{array}{ll} \int f(x, y) & \text{cts} \\ f(x, y) & \text{dis} \end{array} \right. \ldots \)

15. \( E(g(X, Y)) = \left\{ \begin{array}{ll} \ldots & \text{cts} \\ \ldots & \text{dis} \end{array} \right. \ldots \)
16. \( f_{X|Y}(x|y) = \quad \) \( F_{X|Y}(x|y) = P(\quad ) \)

17. \( E(c) = \quad , \) \( E(ag(X, Y) + bh(X, Y)) = \quad \)

18. \( \text{Cov}(X, Y) = \quad , \) \( V(aX + bY) = \quad \), \( \rho(X, Y) = \quad \)

19. \( E(X|Y = y) = \quad \) \( E(E(X|Y)) = \quad , \) \( E(V(X|Y)) + V(E(X|Y)) = \quad \)

20. \( U = h(Y) , \) where \( h \) is \quad \text{and} \ Y \) is cts. \( f_U(u) = f_Y(\quad ) \cdot \quad \)

21. \( \bar{Y} = \quad , \) \( S^2 = S_Y^2 = \quad \)

22. \( F_Y(t) \quad \Phi(\quad ) , \) where \( \Phi(t) \) is the cdf of \quad .

\( X_i \)'s \sim: \quad X_1 + X_2 \sim: \quad G(\alpha_1 + \alpha_2, \beta) \)
\( G(\alpha_i, \beta) \quad \chi^2(v_i + v_2) \)
\( \chi^2(v_i) \quad \text{key: } \perp \quad Pois(\lambda_i + \lambda_2) \)
\( Pois(\lambda_i) \quad N(\mu_i, \sigma^2_i) \quad N(\mu_1 + \mu_2, \sigma^2_1 + \sigma^2_2) \)
\( N(\mu_i, \sigma^2_i) \quad bin(n_i, p) \quad bin(n_1 + n_2, p) \)
MATH 450, ACTUARIAL MATHEMATICS I

The course is a preparation for Long-Term Actuarial Mathematics Exam.

MWF 2:20 - 3:20 CW-115
T 1:15 - 2:40 CW-107

Professor: Qiqing Yu
Office: WH 132
Office hours: 11:00 am - 12:00(M) 3:30-4:30pm (Tu),

Textbook: Arcones’ Manual For SOA Exam MLC (First Volumn).

(Chapters to be covered: 2-6)

A pdf file with some tables needed in the homework can be downloaded from my website.
http://www.math.binghamton.edu/qyu/qyu_personal
e.g, the Illustrative Life Table needed in some of the homework problems.

Exams: 3 tests + final,
Feb. 18(M), Mar. 25(M), April 22(M), May17(F) 12:50-2:50pm CW 106 (moved to WH 100E)

You can bring a calculator without the function of installing formulas.

Quizes: once a week, at random.

Homework: Due Wednesday in class, no late homework.

Grading Policy:

1. 10% hw +10% quiz +45% tests +35% final

2. Correction: If you make correction and hand in with the old exam, within 3 days after I return the test in class, you can get 40% of the missing grades back. No partial credit for correction.

3. A- = 85 + and C = 60 +.

Student Attendance in Class: The Bulletin states, Students are expected to attend all scheduled classes, laboratories and discussions. Instructors may establish their own attendance criteria for a course. They may establish both the number of absences permitted to receive credit for the course and the number of absences after which the final grade may be adjusted downward. In such cases it is expected that the instructor stipulate such requirements in the syllabus and that the syllabus be made available to students at or near the beginning of classes. In the absence of such statements, instructors have the right to deny a student the privilege of taking the final examination or of receiving credit for the course, or may prescribe other academic penalties if the student misses more than 25 percent of the total class sessions. Excessive tardiness may count as absence.
CHAPTER 2
Survival models

2.1 Survival models.

2.1.1 A short probability review.

Definition 2.1. Given a set $\Omega$, a probability $P$ on $\Omega$ is a function defined on the collection of all events (subsets) of $\Omega$ such that

(i) $P(\emptyset) = 0$;
(ii) $P(\Omega) = 1$;
(iii) If $\{A_n\}_{n=1}^\infty$ are disjoint events, then $P(\cup_{n=1}^\infty A_n) = \sum_{n=1}^\infty P(A_n)$.

$\Omega$ is called the sample space.

Definition 2.2. A random variable (r.v.) $X$ is a function from the sample space $\Omega$ into $\mathbb{R}$.

Definition 2.3. The cumulative distribution function (cdf) of the r.v. $X$ is $F_X(x) = P\{X \leq x\}$, $x \in \mathbb{R}$.

Let $X$ be the age at the death of a life. Then $X > 0$.

Theorem 2.1. $F_X$ is a cdf iff

(i) $F_X \uparrow$, i.e., for each $x_1 \leq x_2$, $F_X(x_1) \leq F_X(x_2)$.
(ii) $F_X$ is right continuous (cts) ($\lim_{h \to 0^+} F_X(x+h) = F_X(x)$ $\forall x$)
    (or $F(x+) = F(x)$ $\forall x$).
(iii) $\lim_{x \to -\infty} F_X(x) = 0$ and $\lim_{x \to \infty} F_X(x) = 1$.

For the c.d.f. of an age–at–failure, we only need to define it for $x > 0$ Why ??

Theorem 2.2.

Definition 2.4. A r.v. $X$ is called discrete
if there is a countable set $C \subset \mathbb{R}$ such that $P\{X \in C\} = 1$.

Meaning of countable set ?

Ans. $C$ is either a finite set or $C = \{c_i : i = 1, ..., \infty\}$.

Definition 2.5. The probability mass function (or frequency function) (p.m.f.) of the discrete r.v. $X$ is the function $p : \mathbb{R} \to \mathbb{R}$ defined by

$p(x) = P\{X = x\}$, $x \in \mathbb{R}$.

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