Announcement.
1. Homework due Wenesday after Thanksgiving.
2. Correction to Exam due 3 days after I email you your grade.
3. Quiz formulas all 1-19.
Week 13.

p.429_2. 62. (i) $(I\ddot{a})_x = \sum_{k=0}^{\infty} kv^k \cdot {}_kp_x = \sum_{k=0}^{\infty}(k+1)e^{-0.01k}e^{-0.04k} =$
$\to$
$(I\ddot{a})_x = \sum_{k=0}^{\infty}(k+1)v^k \cdot {}_kp_x =$

Addtional A1: Suppose that $i = 0.05$, the life has a uniform distribution on $(0, 100)$. Find (A) $Var({}_{30}|Y_{20})$ and (B) $Var(\overline{Y}_{\overline{20:30|}})$.

Sol. (A). $Var({}_{30}|Y_{20}) = 2.798506$ and $Var(\overline{Y}_{\overline{20:30|}})$. $= 2.955505$

$$Var({}_{30}|Y_{20}) = E(({}_{30}|Y_{20})^2) - (E({}_{30}|Y_{20}))^2 = 5.979077 - (1.783416)^2 = 2.798506$$

$$\textbf{Reason:}\quad \begin{aligned} E({}_{30}|Y_{20}) =& E(\sum_{k=31}^{K_x-1} v^k) & (1)\\ =& v^{31}E(\sum_{j=0}^{K_x-32} v^j) & j = k-31\\ =& v^{31}E(\frac{1-(v^{K_x-31})}{1-v}I(K_x>31))\\ =& E(\frac{v^{31}-v^{K_x}}{1-v}I(K_x>31))\\ =& \frac{v^{31}{}_{31}p_x - E(v^{K_x}I(K_x>31))}{1-v} & {}_{31}p_x = 1-31/80 \end{aligned}$$

$$\begin{aligned} E(v^{K_x}I(K_x>31)) =& \sum_{k=32}^{80} v^k f_{K_x}(k) = \sum_{k=32}^{80} v^k\frac{1}{80} = v^{32}\sum_{j=0}^{80-32} v^j/80 = v^{32}\frac{1-v^{80-32+1}}{1-v}\frac{1}{80}\\ =& \frac{v^{32}-v^{81}}{1-v}\frac{1}{80} & (2)\\ E(({}_{30}|Y_{20})^2) =& E(\frac{v^{62}-2v^{31}(v^{K_x})+(v^{2K_x})}{(1-v)^2}I(K_x>31))\\ =& \frac{v^{62}{}_{31}p_x - 2v^{31}E(v^{K_x}I(K_x>31)) + E((v^2)^{K_x}I(K_x>31))}{(1-v)^2} \quad (see\ (2)) & (3) \end{aligned}$$

$$Var({}_{30}|Y_{20}) = E(({}_{30}|Y_{20})^2) - (E({}_{30}|Y_{20}))^2 = 5.979077 - (1.783416)^2 = 2.798506 \quad by\ (1),(2),(3)$$

$$\begin{aligned} (B)\quad \overline{Y}_{\overline{20:30|}} =& \int_0^{(T_x\vee n)} v^t dt = \int_0^n v^t dt + \int_n^{T_x} v^t dt I(T_x>n) = \frac{1-v^t}{-\ln v}\Big|_0^n + \frac{1-v^t}{-\ln v}\Big|_n^{T_x} I(T_x>n)\\ =& \frac{1-v^n}{-\ln v} + \frac{v^n - v^{T_x}}{-\ln v}I(T_x>n)\\ =& \frac{1-v^n}{-\ln v} + {}_n|\overline{Y}_x \quad (formula\ [17]). \end{aligned}$$

$$V(\overline{Y}_{\overline{20:30|}}) = V({}_n|\overline{Y}_x) = E(({}_n|\overline{Y}_x)^2) - (E({}_n|\overline{Y}_x))^2 = 6.396208 - (1.854913)^2 = 2.955505 \quad (see\ below).$$

$$E({}_n|\overline{Y}_x) = E(\frac{v^n - v^{T_x}}{-\ln v}I(T_x>n)) = \frac{v^n{}_np_x - E(v^{T_x}I(T_x>n))}{-\ln v} \qquad (see\ (4)\ below)$$

$$E(v^{T_x}I(T_x>n)) = \frac{v^t}{\ln v}\Big|_n^{80}/80 = \frac{v^n - v^{80}}{-80\ln v} \qquad (4)$$

$$E(({}_n|\overline{Y}_x)^2) = E((\frac{v^n - v^{T_x}I(T_x>n)}{-\ln v})^2) = \frac{v^{2n}{}_np_x - 2v^nE(v^{T_x}I(T_x>n)) + E((v^2)^{T_x}I(T_x>n))}{(-\ln v)^2} \qquad (see\ (4))$$

```
v=(1/1.05)
a=(v**32-v**81)/(1-v)/80
e=(v**31*(1-31/80)-a)/(1-v) [1] 1.783416
b=v**62*(1-31/80)-2*v**31*a
v=v*v
a=(v**32-v**81)/(1-v)/80
b=b+a
v=(1/1.05)
b=b/(1-v)**2 [1] 5.979077
b-e*e [1] 2.798506
```

```
v=1/1.05
n=30
a=(v**n-v**80)/(-80*log(v))
e=(v**n*(1-n/80)-a)/(-log(v)) [1] 1.854913
b=v**(2*n)*(1-n/80)-2*v**n*a
v=v*v
a=(v**n-v**80)/(-80*log(v))
v=1/1.05
b=(b+a)/(log(v))**2 [1] 6.396208
b-e*e [1] 2.955505
```