

Formulas For Math 447

Part A. Fill the blanks of the formulas (30 points)

1. DeMorgan's laws: $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$, and $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$.
2. Axioms of probability: (1) $P(A) \geq \underline{0}$, (2) $P(S) = \underline{1}$,
(3) if A_i 's are pairwise disjoint, then $P(\cup_i A_i) = \sum_i P(A_i)$.
3. If each sample point in the sample space S is equally likely to occur, $P(A) = \frac{N(\underline{\quad})}{N(\underline{\quad})}$ **key:**
A, S
4. If an experiment can be partitioned into 2 steps, with n_i choices in the i^{th} step, then the total # of choices in the experiment is $n_1 n_2$
5. $P_r^n = \frac{n!}{(n-r)!}$, $C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$,
 $\binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1! n_2! n_3!}$, where $n_1 + n_2 + n_3 = \underline{n}$
6. $P(A \cap B) = \underline{P(A|B)P(B)}$. If A and B are independent then $P(A \cap B) = P(A)P(B)$.
7. $P(\overline{A}) = 1 - P(A)$. $P(A \cup B) = P(A) + P(B) - \underline{P(A \cap B)}$,
If A and B are disjoint then $P(A \cup B) = \underline{P(A) + P(B)}$.
8. Bayes' Rule: If $\{B_1, \dots, B_k\}$ is a partition of S and $P(A), P(B_j) > 0$, then
$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{k=1}^k P(A|B_k)P(B_k)}$$
9. $X \sim \text{bin}(n, p)$: $f(i) = \underline{\binom{n}{i} p^i q^{n-i}}$, if $i \in \{0, \dots, n\}$, $\mu = \underline{np}$, $\sigma^2 = \underline{npq}$ where $q = 1 - p$
10. $X \sim G(p)$. $f(i) = \underline{q^{i-1} p}$ if $i \geq \underline{1}$, $\mu = \underline{1/p}$, $\sigma^2 = \underline{q/p^2}$
11. $X \sim \text{Nbin}(r, p)$, $f(i) = \underline{\binom{i-1}{r-1} p^r q^{i-r}}$ if $i \geq \underline{r}$, $\mu = \underline{r/p}$, $\sigma^2 = \underline{rq/p^2}$
12. $X \sim \text{HG}(N, n, r)$. $f(i) = \underline{\frac{\binom{r}{i} \binom{N-r}{n-i}}{\binom{N}{n}}}$, $\mu = \underline{nr/N}$, $\sigma^2 = \underline{\frac{nr(N-r)(N-n)}{N^2(N-1)}}$.
13. $X \sim \text{Pois}(\lambda)$. $f(i) = \underline{\frac{e^{-\lambda} \lambda^i}{i!}}$, if $i \geq 0$. $\mu = \underline{\lambda}$, $\sigma^2 = \underline{\lambda}$
14. Tchebysheff's Inequality. $P(|X - \mu| > k\sigma) \leq \underline{1/k^2}$
15. $Y = g(X)$. $E(g(X)) = \begin{cases} \sum_y y f_Y(y) & \text{dis} \\ \int y f_Y(y) dy & \text{cts} \end{cases} = \begin{cases} \sum_x g(x) f_X(x) & \text{dis} \\ \int g(x) f_X(x) dx & \text{cts} \end{cases}$, $\mu_Y = \underline{E(Y)}$,
$$\sigma_Y^2 = \underline{E(Y^2) - \mu_Y^2}$$
16. The mgf of X is $M(t) = \underline{E(e^{Xt})}$, $\frac{d^k M(t)}{dt^k} \Big|_{t=0} = \underline{E(X^k)}$
17. A cdf $F(t)$ ($= P(X \leq t)$,) satisfying (1) $F(-\infty) = \underline{0}$, and $F(\infty) = \underline{1}$, (2) $F(x+) = \underline{F(x)}$,
(3) $F(x) \uparrow$. Moreover, $F(b) - F(a) = \underline{P(a < X \leq b)}$
18. $F(t) = \begin{cases} \sum_{x \leq t} \underline{f(x)} & \text{dis} \\ \int_{-\infty}^t \underline{f(x)} dx & \text{cts} \end{cases}$, $f(t) = \begin{cases} \underline{F(t) - F(t-)} & \text{dis} \\ \underline{F'(t)} & \text{cts} \end{cases}$
19. For discrete r.v. X , the p.f. f satisfies $f(t) = P(\underline{X = t})$, $f(t) \in \underline{[0, 1]}$, and $\sum_t f(t) = \underline{1}$,
For cts r.v. X , the d.f. f satisfies $f(t) \geq \underline{0}$, and $\int f(t) dx = \underline{1}$
20. $E(aX + b) = \underline{aE(X) + b}$, $\text{Var}(aX + b) = \underline{a^2 \sigma_X^2}$
21. $X \sim U(a, b)$. $f(x) = \underline{\frac{1}{b-a}}$, if $x \in (a, b)$. $\mu = \underline{\frac{a+b}{2}}$, $\sigma^2 = \underline{\frac{(b-a)^2}{12}}$
22. $X \sim N(\mu, \sigma^2)$. $f(x) = \underline{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$, $\frac{X-\mu}{\sigma} \sim \underline{N(0, 1)}$
23. $X \sim \mathcal{G}(\alpha, \beta)$. $f(x) = \underline{\frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}}$, if $x > \underline{0}$, $\mu = \underline{\alpha\beta}$, $\sigma^2 = \underline{\alpha\beta^2}$, $\Gamma(\alpha + 1) = \underline{\alpha\Gamma(\alpha)}$
24. $\text{Exp}(\lambda) = \underline{\mathcal{G}(1, \lambda)}$, $\chi^2(\nu) = \underline{\mathcal{G}(\frac{\nu}{2}, 2)}$

25. $X \sim \text{beta}(\alpha, \beta)$. $f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$, if $x \in (0, 1)$, $\mu = \frac{\alpha}{\alpha+\beta}$, where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$
26. The cdf of (X, Y) is $F(x, y) = P(X \leq x, Y \leq y) = \begin{cases} \int_{-\infty}^x \int_{-\infty}^y f(t, s) ds dt & \text{cts} \\ \sum_{t \leq x} \sum_{s \leq y} f(t, s) & \text{dis} \end{cases}$
27. For continuous (X, Y) , their d.f. $f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$, $f(x, y) \geq 0$, $\int \int f(x, y) dx dy = 1$,
For discrete (X, Y) , their d.f. $f(x, y) = P\{X = x, Y = y\}$, $f(x, y) \in [0, 1]$,
 $\sum_x \sum_y f(x, y) = 1$
28. $P((X, Y) \in A) = \begin{cases} \frac{\int \int_{(x, y) \in A} f(x, y) dx dy}{\sum_{(x, y) \in A} f(x, y)} & \text{cts} \\ \sum_{(x, y) \in A} f(x, y) & \text{dis} \end{cases}$
29. $f_X(x) = \begin{cases} \int f(x, y) dy & \text{cts} \\ \sum_y f(x, y) & \text{dis} \end{cases}$
30. $f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)}$, $F_{X|Y}(x|y) = P(X \leq x | Y = y)$
31. $E(g(X, Y)) = \begin{cases} \int \int g(x, y) f(x, y) dx dy & \text{cts} \\ \sum_x \sum_y g(x, y) f(x, y) & \text{dis} \end{cases}$
32. The cts or discrete X and Y are independent iff $F(x, y) = F_X(x)F_Y(y)$ iff $f(x, y) = f_X(x)f_Y(y)$
33. $E(c) = c$, $E(ag(X, Y) + bh(X, Y)) = aE(g(X, Y)) + bE(h(X, Y))$
34. $Cov(X, Y) = E(XY) - E(X)E(Y)$, $V(aX + bY) = a^2V(X) + b^2V(Y) + 2abCov(X, Y)$,
 $\rho = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$,
35. $(Y_1, Y_2, \dots, Y_k) \sim M(n, p_1, \dots, p_k)$. $f(y_1, \dots, y_k) = \frac{\binom{n}{y_1, \dots, y_k}}{p_1^{y_1} \dots p_k^{y_k}}$, $y_i \in \{0, 1, \dots, n\}$,
 $\sum_i p_i = 1$, and $\sum_i y_i = n$, $Cov(Y_i, Y_j) = -np_i p_j$, if $i \neq j$, $Y_i \sim \text{bin}(n, p_i)$
36. $E(X|Y = y) = \begin{cases} \frac{\int x f(x|y) dx}{\sum_x x f(x|y)} & \text{cts} \\ \sum_x x f(x|y) & \text{dis} \end{cases}$. $E(E(X|Y)) = E(X)$,
 $E(V(X|Y)) + V(E(X|Y)) = V(X)$
37. $U = h(Y_1, \dots, Y_n)$, where Y_i 's are continuous. $F_U(t) = \int \dots \int_A f(y_1, \dots, y_n) dy_1 \dots dy_n$, where
 $A = \{h(y_1, \dots, y_n) \leq t\}$, $f_U(t) = F'_U(t)$
38. $U = h(Y)$, where h is monotone, and Y is cts. $f_U(u) = f_Y(h^{-1}(u)) \cdot \left| \frac{\partial h^{-1}}{\partial u} \right|$
39. If X and Y are independent, then $Cov(X, Y) = 0$, $E(g(X)h(Y)) = E(g(X))E(h(Y))$
 $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$
- Hereafter, let Y_1, \dots, Y_n be a random sample of Y .**
40. Let Y_1, \dots, Y_n be a random sample of Y . $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$, $S^2 = S_Y^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2$
41. If $Y \sim N(\mu, \sigma^2)$, $\frac{\bar{Y} - \mu_{\bar{Y}}}{\sigma_{\bar{Y}}} \sim N(0, 1)$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, $\sqrt{n} \frac{\bar{Y} - \mu}{S} \sim t_{n-1}$, where $\mu_{\bar{Y}} = \mu$,
 $\sigma_{\bar{Y}}^2 = \sigma^2/n$
42. $F_{\bar{Y}}(t) \approx \Phi\left(\frac{t - \mu_{\bar{Y}}}{\sigma_{\bar{Y}}}\right)$, where $\Phi(t)$ is the cdf of $N(0, 1)$

Review formulas for Math 448,

1. Estimator of μ is \bar{X} where $\bar{X} = \frac{\sum_i X_i}{n}$, Estimator of σ^2 is S^2 , where $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$. **key:** $\sum_i X_i/n, \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$.
2. An estimator $\hat{\theta}$ is unbiased if $E(\hat{\theta}) = \theta$, bias $B(\hat{\theta}) = E(\hat{\theta}) - \theta$, MSE = $V(\hat{\theta}) + (B(\hat{\theta}))^2$. **(key: $E(\hat{\theta}) = \theta, E(\hat{\theta}) - \theta, V(\hat{\theta}) + (B(\hat{\theta}))^2$),**
3. $100(1 - \alpha)\%$ CI for θ : $\hat{\theta} \pm z_{\alpha/2} \hat{\sigma}_{\hat{\theta}}$. **key: $P(a \leq \theta \leq b), 1 - \alpha$.**
4. $100(1 - \alpha)\%$ large sample CI for θ : $\hat{\theta} \pm z_{\alpha/2} \hat{\sigma}_{\hat{\theta}}$. **key: $\hat{\theta} \pm z_{\alpha/2} \hat{\sigma}_{\hat{\theta}}$.**
5. If (1) X_1, \dots, X_n are i.i.d. from $N(\mu_x, \sigma_x^2)$, (2) Y_1, \dots, Y_m are i.i.d. from $N(\mu_y, \sigma_y^2)$, and (3) X_i 's \perp Y_j 's, then $T = \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{S_x/\sqrt{n}}$, $\sim \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{\hat{\sigma}_p \sqrt{1/n_x + 1/n_y}}$, where $\hat{\sigma}_p = \frac{\sqrt{S_x^2 + S_y^2}}{\sqrt{2}}$, $W = (n_x - 1)S_x^2/\sigma_x^2 \sim \chi_{n_x-1}^2$, $F = S_x^2/S_y^2 \sim F_{n_x-1, n_y-1}$. **key: $\sigma_x^2, \sigma_y^2, \perp, t_{n_x-1}, t_{n_y-1}, \sqrt{\frac{(n_x-1)S_x^2 + (n_y-1)S_y^2}{n_x+n_y-2}}, \chi_{n_x-1}^2, F_{n_x-1, n_y-1}$,**
6. $f_{Y_{(j)}}(t) = \binom{n}{j-1, 1, n-j} (F(t))^{j-1} (f(t)) (1 - F(t))^{n-j}$. **key: $(F(t))^{j-1} (f(t)) (1 - F(t))^{n-j}$.**
7. Given two unbiased estimators $\hat{\theta}_1$ and $\hat{\theta}_2$, $\text{eff}(\hat{\theta}_1, \hat{\theta}_2) = \frac{V(\hat{\theta}_2)}{V(\hat{\theta}_1)}$. **key: unbiased, $\frac{V(\hat{\theta}_2)}{V(\hat{\theta}_1)}$,**
8. $\hat{\theta}$ is consistent or $\hat{\theta} \xrightarrow{P} \theta$ if $P(|\hat{\theta} - \theta| > \epsilon) \rightarrow 0$, $\forall \epsilon > 0$. **key: $0, \forall \epsilon$,**
9. If (1) $E(\hat{\theta}) = \theta$ and (2) $V(\hat{\theta}) \rightarrow 0$, as $n \rightarrow \infty$, then $\hat{\theta}$ is consistent. **key: $\theta, \rightarrow 0$,**
10. If $g(\cdot)$ is continuous, $\hat{\theta}_1 \xrightarrow{P} \theta_1$ and $\hat{\theta}_2 \xrightarrow{P} \theta_2$, then $\hat{\theta}_1 \pm \hat{\theta}_2 \xrightarrow{P} \theta_1 \pm \theta_2$, $\hat{\theta}_1 \hat{\theta}_2 \xrightarrow{P} \theta_1 \theta_2$, $g(\hat{\theta}_1) \xrightarrow{P} g(\theta_1)$. **key: cts, $\theta_1 \pm \theta_2, \theta_1 \theta_2, g(\theta_1)$,**
11. If X_1, \dots, X_n are i.i.d. from $f(x; \theta) = \exp\{T(x)\psi(\theta) + g(\theta) + h(x)\}$, $\hat{\gamma} = G(\frac{\sum_i T(X_i)}{n})$ and $\gamma = \gamma(\theta)$, then $\hat{\gamma}$ is the MVUE of γ . **key: $T(x)\psi(\theta), \sum_i T(X_i), E(\hat{\gamma})$,**
12. An MME of θ , is the solution of θ to $\mu'_k(\theta) = \bar{X}_k$ for k 's, where $\mu'_k(\theta) = \frac{\partial \mu_k(\theta)}{\partial \theta}$, and k is the dimension of θ . **key: $\bar{X}_k, E(X^k)$,**
13. Given a random sample X_1, \dots, X_n from $f(x; \theta)$, their likelihood is $L(\theta) = \prod_i f(X_i; \theta)$, the MLE $\hat{\theta}$ of θ maximizes $L(\theta)$. If $g(\theta)$ is a function of θ , the MLE of $g(\theta)$ is $g(\hat{\theta})$. **key: $\prod_i f(X_i; \theta), L(\theta), 1 - 1, g(\hat{\theta})$,**
14. Given a random sample from $f(x; \theta)$, and a function $g(\theta)$, under certain conditions, $P(g(\hat{\theta}) \leq t) \approx \Phi(\frac{t - g(\hat{\theta})}{\hat{\sigma}_{g(\hat{\theta})}})$, where $\hat{\theta}$ is the MLE of θ , $\hat{\sigma}_{g(\hat{\theta})} = \frac{1}{\sqrt{n}} \sqrt{\left\{ \left(\frac{\partial g}{\partial \theta} \right)^2 / \left(-n E \left(\frac{\partial^2 \log f(X; \theta)}{\partial \theta^2} \right) \right) \right\} |_{\theta = \hat{\theta}}}$. **key: $\approx, MLE, \sqrt{\left\{ \left(\frac{\partial g}{\partial \theta} \right)^2 / \left(-n E \left(\frac{\partial^2 \log f(X; \theta)}{\partial \theta^2} \right) \right) \right\} |_{\theta = \hat{\theta}}}$,**
15. The 5 elements of a test are (1) H_0 , (2) H_a , (3) test statistics (4) α , (5) β . **key: $H_0, H_a, RR, Conclusion$,**
16. Probability of type I error is α , Probability of type II error is β . **key: $P(H_a|H_0), P(H_0|H_a)$,**
17. For a large sample test for $H_0: \theta = \theta_0$, a test statistic is $Z = \frac{\hat{\theta} - \theta_0}{\hat{\sigma}_{\hat{\theta}}}$, a RR is $Z > z_{\alpha/2}$ if $\theta > \theta_0$; and a RR is $Z < -z_{\alpha/2}$ if $\theta < \theta_0$; **key: $\frac{\hat{\theta} - \theta_0}{\hat{\sigma}_{\hat{\theta}}}, > z_{\alpha/2}, |Z| > z_{\alpha/2}$,**

18. Sample size for an upper-tail α -level test is $n = (\text{_____})^2$ **key:** $\frac{(z_\alpha + z_\beta)\sigma}{\mu_a - \mu_o}$,

19. The P-value is $\begin{cases} P(W \text{_____} w) | H_o \text{ is correct} & \text{if } H_a : \theta > \theta_o \\ P(W \text{_____} w) | H_o \text{ is correct} & \text{if } H_a : \theta < \theta_o \\ \text{_____} P(W \text{_____} |w|) | H_o \text{ is correct} & \text{if } H_a : \theta \neq \theta_o \end{cases}$ where W is the (Z or T) test statistic and w is the observed value of W . **key:** $\geq, \leq, \underline{\geq}, \underline{\leq}$,

20. Suppose that $Z \sim N(0, 1)$, $X \sim \chi^2(u)$, $Y \sim \chi^2(v)$. If $Z \text{_____} X$, $T = \text{_____}$, then $T \sim t_u$; If $X \text{_____} Y$, $F = \text{_____}$, then $F \sim F_{u,v}$ and $X + Y \sim \text{_____}$. **key:** $\underline{\perp}$, $\underline{Z/\sqrt{X/u}}$, $\underline{\perp}$, $\underline{X/u}$, $\underline{Y/v}$, $\underline{\chi^2(u+v)}$,

21. The MP test for $H_o: \theta = \theta_o$ v.s. $H_a: \theta = \theta_a$, the MP test has the RR satisfying: $\frac{L(\theta_o)}{L(\theta_a)} \text{_____} k$ and $P_\theta(RR) = \alpha$ if $\theta = \text{_____}$. **key:** $\leq, \underline{\theta_o}$,

22. The Likelihood ratio test for $H_o: \theta \in \Theta_o$ v.s. $H_a: \theta \notin \Theta_o$ has a RR: $\{\lambda \text{_____} k\}$, where

$\lambda = \text{_____}$; $\hat{\theta}_o$ is the MLE under _____ ; $\hat{\theta}$ is the MLE under _____ ;

k satisfies $\max\{P(RR) : \theta \in \Theta_o\} = \text{_____}$; if n is large, then $-2\ln\lambda$ is approximated _____ ; where $v = \text{_____}$; r and $r_o = \#$ of free parameters in Θ and in Θ_o , respectively.

key: $\underline{\leq}$, $\underline{\frac{L(\hat{\theta}_o)}{L(\hat{\theta})}}$, $\underline{\Theta_o}$, $\underline{\Theta}$, $\underline{\alpha}$, $\underline{\chi^2(v)}$, $\underline{r - r_o}$

23. Suppose (x_i, Y_i) 's are independent, $\mu_{x_i}(\theta) = E(Y_i)$, the LSE of θ , denoted by $\hat{\theta}$, minimizes _____ .

If $\mu_{x_i}(\theta) = \beta_0 + \beta_1 x_i$ and $V(Y_i) = \sigma^2$, the LSE $\hat{\beta}_0 = \text{_____}$,

$\hat{\beta}_1 = \text{_____}$, $S_{xy} = \text{_____}$, $E(\hat{\beta}_0) = \text{_____}$, $E(\hat{\beta}_1) = \text{_____}$,

$\sigma_{\hat{\beta}_0}^2 = \text{_____}$, $\sigma_{\hat{\beta}_1}^2 = \text{_____}$, an unbiased estimator of σ^2 is $\hat{\sigma}^2 = \text{_____}$,

key: $\underline{\sum_{i=1}^n [Y_i - \mu_{x_i}(\theta)]^2}$, $\underline{\bar{Y} - \hat{\beta}_1 \bar{x}}$, $\underline{S_{xy}/S_{xx}}$, $\underline{n(\bar{x}\bar{Y} - \bar{x}\bar{Y})}$, $\underline{\beta_0}$, $\underline{\beta_1}$,

$\underline{\sigma^2 \bar{x}^2 / S_{xx}}$, $\underline{\sigma^2 / S_{xx}}$, $\underline{\frac{1}{n-2}(S_{yy} - \hat{\beta}_1 S_{xy})}$,

If $Y_i \sim \text{_____}$, then $\hat{\beta}_j \sim \text{_____}$, $\frac{n-2}{\sigma^2} \hat{\sigma}^2 \sim \text{_____}$, $\hat{\sigma}^2, \hat{\beta}_0, \hat{\beta}_1$ are _____ , $\text{_____} \sim t_{n-2}$,

key: $\underline{N(\beta_0 + \beta_1 x_i, \sigma^2)}$, $\underline{N(\beta_j, \sigma_{\hat{\beta}_j}^2)}$, $\underline{\chi^2(n-2)}$, $\underline{\text{independent}}$, $\underline{\frac{\hat{\beta}_j - \beta_j}{\hat{\sigma}_{\hat{\beta}_j}}}$,

24. Under the Bayes model, conditional on θ , Y_1, \dots, Y_n are i.i.d. with $f(y|\theta)$, and $\theta \sim g(\theta)$. The posterior df is $g(\theta|y) = \underline{\hspace{10em}}$, where $y = (y_1, \dots, y_n)$ the Bayes estimator of $h(\theta)$ is $\hat{h} = \underline{\hspace{10em}}$

key: $\frac{\prod_{i=1}^n f(y_i|\theta)g(\theta)}{f_Y(y)}, E(h(\theta)|y)$,

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23. $X \sim \mathcal{G}(\alpha, \beta)$. $f(x) = \underline{\hspace{10em}}$ if $x > \underline{\hspace{2em}}$, $\mu = \underline{\hspace{2em}}$, $\sigma^2 = \underline{\hspace{2em}}$, $\Gamma(\alpha + 1) = \underline{\hspace{2em}}$, **key:** $\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}, 0, \alpha\beta, \alpha\beta^2, \alpha\Gamma(\alpha)$

24. $Exp(\lambda) = \underline{\hspace{10em}}$, $\chi^2(\nu) = \underline{\hspace{10em}}$, **key:** $\mathcal{G}(1, \lambda), \mathcal{G}(\frac{\nu}{2}, 2)$

25. $X \sim beta(\alpha, \beta)$. $f(x) = \underline{\hspace{10em}}$ if $x \underline{\hspace{2em}}$, $\mu = \underline{\hspace{2em}}$, where $B(\alpha, \beta) = \underline{\hspace{2em}}$, **key:** $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}, \in (0, 1), \frac{\alpha}{\alpha+\beta}, \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

41. If Y_i 's are i.i.d. with $N(\mu, \sigma^2)$, then $\frac{\bar{Y} - \mu_{\bar{Y}}}{\sigma_{\bar{Y}}} \sim \underline{\hspace{2em}}$, $\frac{(n-1)S^2}{\sigma^2} \sim \underline{\hspace{2em}}$, $S^2 \underline{\hspace{2em}} \bar{Y}$, where $\mu_{\bar{Y}} = \underline{\hspace{2em}}$, $\sigma_{\bar{Y}}^2 = \underline{\hspace{2em}}$ **key:** $N(0, 1), \chi^2(n-1), \perp, \mu, \sigma^2/n$

	X_i 's \sim :	$X_1 + X_2 \sim$:	
	$\mathcal{G}(\alpha_i, \beta)$	$\underline{\hspace{2em}}$	$\frac{\mathcal{G}(\alpha_1 + \alpha_2, \beta)}{\chi^2(v_1 + v_2)}$
	$\chi^2(v_i)$	$\underline{\hspace{2em}}$	$\frac{Pois(\lambda_1 + \lambda_2)}{N(\mu_x + \mu_y, \sigma_1^2 + \sigma_2^2)}$
44. If $X_1 \underline{\hspace{2em}} X_2$.	$Pois(\lambda_i)$	$\underline{\hspace{2em}}$	$\frac{bin(n_1 + n_2, p)}{\underline{\hspace{2em}}}$
	$N(\mu_i, \sigma_i^2)$	$\underline{\hspace{2em}}$	
	$bin(n_i, p)$	$\underline{\hspace{2em}}$	