Formulas For Math 447

Part A. Fill the blanks of the formulas (30 points)

1. DeMorgan's laws: $\overline{(A\cap B)} = \underline{\overline{A}\cup\overline{B}}$, and $\overline{(A\cup B)} = \underline{\overline{A}\cap\overline{B}}$.
2. Axioms of probability: (1) $P(A)\geq \underline{0}$, (2) $P(S)=\underline{1}$,
   (3) if A_i's are pairwise $\underline{disjoint}$, then $P(\cup_i A_i)=\sum_i P(A_i)$.
3. If each sample point in the sample space S is equally likely to occur, $P(A)=\frac{N(\_\_\_\_)}{N(\_\_\_\_)}$ **key:** $\underline{A}$, $\underline{S}$
4. If an experiment can be partitioned into 2 steps, with n_i choices in the i^{th} step, then the total # of choices in the experiment is $\underline{n_1 n_2}$
5. $P^n_r = \underline{\frac{n!}{(n-r)!}}$, $C^n_r = \binom{n}{r} = \underline{\frac{n!}{r!(n-r)!}}$,
   $\binom{n}{n_1,n_2,n_3} = \underline{\frac{n!}{n_1!n_2!n_3!}}$, where $n_1+n_2+n_3=\underline{n}$
6. $P(A\cap B) = \underline{P(A|B)P(B)}$. If A and B are $\underline{independent}$ then $P(A\cap B)=P(A)P(B)$.
7. $P(\ \underline{\overline{A}}\ ) = 1-P(A)$. $P(A\cup B)=P(A)+P(B)\ \underline{-P(A\cap B)}$,
   If A and B are $\underline{disjoint}$ then $P(A\cup B)=\underline{P(A)+P(B)}$.
8. Bayes' Rule: If $\underline{\{B_1,...,B_k\}}$ is a partition of S and $P(A), P(B_j)>0$, then
   $P(B_j|A) = \underline{\frac{P(A|B_j)P(B_j)}{\sum_{k=1}^{k} P(A|B_k)P(B_k)}}$
9. $X\sim bin(n,p)$: $f(i)=\underline{\binom{n}{i}p^i q^{n-i}}$, if $i\in\{0,...,n\}$, $\mu=\underline{np}$, $\sigma^2=\underline{npq}$ where $q=1-p$
10. $X\sim G(p)$. $f(i)=q^{i-1}p$ if $i\geq\underline{1}$, $\mu=\underline{1/p}$, $\sigma^2=\underline{q/p^2}$
11. $X\sim Nbin(r,p)$, $f(i)=\binom{i-1}{r-1}p^r q^{i-r}$ if $i\geq\underline{r}$, $\mu=\underline{r/p}$, $\sigma^2=\underline{rq/p^2}$
12. $X\sim HG(N,n,r)$. $f(i)=\underline{\frac{\binom{r}{i}\binom{N-r}{n-i}}{\binom{N}{n}}}$, $\mu=\underline{nr/N}$, $\underline{\sigma^2}=\frac{nr(N-r)(N-n)}{N^2(N-1)}$.
13. $X\sim Pois(\lambda)$. $f(i)=\underline{\frac{e^{-\lambda}\lambda^i}{i!}}$, if $i\geq 0$. $\mu=\underline{\lambda}$, $\sigma^2=\underline{\lambda}$
14. Tchebysheff's Inequality. $P(|X-\mu|>k\sigma)\ \underline{\leq 1/k^2}$
15. $Y=g(X)$. $E(g(X))=\begin{cases}\sum_y y f_Y(y) & \text{dis}\\ \int y f_Y(y)dy & \text{cts}\end{cases} = \begin{cases}\underline{\sum_x g(x)f_X(x)} & \text{dis}\\ \underline{\int g(x)f_X(x)dx} & \text{cts}\end{cases}$, $\mu_Y=\underline{E(Y)}$,
    $\sigma^2_Y = \underline{E(Y^2)-\mu_Y^2}$
16. The mgf of X is $M(t)=\underline{E(e^{Xt})}$, $\frac{d^k M(t)}{dt^k}\Big|_{t=0}=\underline{E(X^k)}$
17. A cdf $F(t)$ ($=\underline{P(X\leq t)}$,) satisfying (1) $F(-\infty)=\underline{0}$, and $F(\infty)=\underline{1}$, (2) $F(x+)=\underline{F(x)}$,
    (3) $F(x)\ \underline{\uparrow}$. Moreover, $F(b)-F(a)=$ P($\underline{a<X\leq b}$)
18. $F(t)=\begin{cases}\sum_{x\leq t}\underline{f(x)} & \text{dis}\\ \int_{-\infty}^{t}\underline{f(x)dx} & \text{cts}\end{cases}$, $f(t)=\begin{cases}\underline{F(t)-F(t-)} & \text{dis}\\ \underline{F'(t)} & \text{cts}\end{cases}$
19. For discrete r.v. X, the p.f. f satisfies $f(t)=P(\ \underline{X=t})$, $f(t)\in\underline{[0,1]}$, and $\sum_t f(t)=\underline{1}$,
    For cts r.v. X, the d.f. f satisfies $f(t)\ \underline{\geq 0}$, and $\int f(t)dx=\underline{1}$
20. $E(aX+b)=\underline{aE(X)+b}$, $Var(aX+b)=\underline{a^2\sigma^2_X}$
21. $X\sim U(a,b)$. $f(x)=\underline{\frac{1}{b-a}}$, if $x\in(a,b)$. $\mu=\underline{\frac{a+b}{2}}$, $\sigma^2=\underline{\frac{(b-a)^2}{12}}$
22. $X\sim N(\mu,\sigma^2)$. $f(x)=\underline{\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$, $\frac{X-\mu}{\sigma}\sim\underline{N(0,1)}$
23. $X\sim\mathcal{G}(\alpha,\beta)$. $f(x)=\underline{\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}}$, if $x>\underline{0}$, $\mu=\underline{\alpha\beta}$, $\sigma^2=\underline{\alpha\beta^2}$, $\Gamma(\alpha+1)=\underline{\alpha\Gamma(\alpha)}$
24. $Exp(\lambda)=\underline{\mathcal{G}(1,\lambda)}$, $\chi^2(\nu)=\underline{\mathcal{G}(\frac{\nu}{2},2)}$

25. $X \sim beta(\alpha,\beta)$. $f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$, if $x \in (0,1)$, $\mu = \frac{\alpha}{\alpha+\beta}$, where $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

26. The cdf of (X,Y) is $F(x,y) = P(X \le x, Y \le y) = \begin{cases} \int_{-\infty}^{x}\int_{-\infty}^{y} f(t,s)dsdt & \text{cts} \\ \sum_{t\le x}\sum_{s\le y} f(t,s) & \text{dis} \end{cases}$

27. For continuous (X,Y), their d.f. $f(x,y) = \frac{\partial^2 F(x,y)}{\partial x \partial y}$, $f(x,y) \ge 0$, $\int\int f(x,y)dxdy = 1$,
For discrete (X,Y), their d.f. $f(x,y) = P\{X = x, Y = y\}$, $f(x,y) \in [0,1]$,
$\sum_x \sum_y f(x,y) = 1$

28. $P((X,Y) \in A) = \begin{cases} \int\int_{(x,y)\in A} f(x,y)dxdy & \text{cts} \\ \sum_{(x,y)\in A} f(x,y) & \text{dis} \end{cases}$

29. $f_X(x) = \begin{cases} \int f(x,y)dy & \text{cts} \\ \sum_y f(x,y) & \text{dis} \end{cases}$

30. $f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}$, $F_{X|Y}(x|y) = P\ (X \le x | Y = y)$

31. $E(g(X,Y)) = \begin{cases} \int\int g(x,y)f(x,y)dxdy & \text{cts} \\ \sum_x \sum_y g(x,y)f(x,y) & \text{dis} \end{cases}$

32. The cts or discrete X and Y are independent iff $F(x,y) = F_X(x)F_Y(y)$ iff $f(x,y) = f_X(x)f_Y(y)$

33. $E(c) = c$, $E(ag(X,Y) + bh(X,Y)) = aE(g(X,Y)) + bE(h(X,Y))$

34. $Cov(X,Y) = E(XY) - E(X)E(Y)$, $V(aX + bY) = a^2V(X) + b^2V(Y) + 2abCov(X,Y)$,
$\rho = \frac{Cov(X,Y)}{\sigma_X\sigma_Y}$,

35. $(Y_1, Y_2, ..., Y_k) \sim M(n, p_1, ..., p_k)$. $f(y_1,...,y_k) = \binom{n}{y_1,...,y_k} p_1^{y_1}\cdots p_k^{y_k}$, $y_i \in \{0,1,...,n\}$,
$\sum_i p_i = 1$, and $\sum_i y_i = n$, $Cov(Y_i,Y_j) = -np_ip_j$, if $i \ne j$, $Y_i \sim bin(n,p_i)$

36. $E(X|Y=y) = \begin{cases} \int xf(x|y)dx & \text{cts} \\ \sum_x xf(x|y) & \text{dis} \end{cases}$. $E(E(X|Y)) = E(X)$,
$E(V(X|Y)) + V(E(X|Y)) = V(X)$

37. $U = h(Y_1,...,Y_n)$, where Y_i's are continuous. $F_U(t) = \int\cdots\int_A f(y_1,...,y_n)dy_1\cdots dy_n$, where
$A = \{h(y_1,...,y_n) \le t\}$, $f_U(t) = F_U'(t)$

38. $U = h(Y)$, where h is *monotone*, and Y is cts. $f_U(u) = f_Y(\ h^{-1}(u)\ )\cdot \left|\frac{\partial h^{-1}}{\partial u}\right|$

39. If X and Y are independent, then $Cov(X,Y) = 0$, $E(g(X)h(Y)) = E(g(X))E(h(Y))$
$M_{X+Y}(t) = M_X(t)\cdot M_Y(t)$
Hereafter, let Y_1, ..., Y_n be a random sample of Y.

40. Let Y_1, ..., Y_n be a random sample of Y. $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, $S^2 = S_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$

41. If $Y \sim N(\mu,\sigma^2)$, $\frac{\overline{Y}-\mu_{\overline{Y}}}{\sigma_{\overline{Y}}} \sim N(0,1)$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, $\sqrt{n}\frac{\overline{Y}-\mu}{S} \sim t_{n-1}$, where $\mu_{\overline{Y}} = \mu$,
$\sigma^2_{\overline{Y}} = \sigma^2/n$

42. $F_{\overline{Y}}(t) \approx \Phi(\ \frac{t-\mu_{\overline{Y}}}{\sigma_{\overline{Y}}}\ )$, where $\Phi(t)$ is the cdf of $N(0,1)$

Review formulas for Math 448,

1. Estimator of μ is $\overline{X}$ where $\overline{X}=$ ______________, Estimator of σ^2 is S^2, where $S^2=$ ______________, **key:** $\underline{\sum_i X_i/n}$, $\underline{\frac{1}{n-1}\sum_{i=1}^n (X_i-\overline{X})^2}$.
2. An estimator $\hat\theta$ is unbiased if ______________, bias $B(\hat\theta)=$ ______________, MSE $=$ __________________, (**key:** $\underline{E(\hat\theta)=\theta}$, $\underline{E(\hat\theta)-\theta}$, $\underline{V(\hat\theta)+(B(\hat\theta))^2}$,
3. $100(1-\alpha)\%$ CI for θ: __________________ $=$ ______ **key:** $\underline{P(a\le\theta\le b)}$, $\underline{1-\alpha}$.
4. $100(1-\alpha)\%$ large sample CI for θ: ______________ **key:** $\underline{\hat\theta\pm z_{\alpha/2}\hat\sigma_{\hat\theta}}$.
5. If (1) X_1, ..., X_n are i.i.d. from $N(\mu_1,\_\_\_)$, (2) Y_1, ..., Y_m are i.i.d. from $N(\mu_2,\_\_\_)$, and (3) X_i's ___ Y_j's, then $T=\underline{\frac{\overline{X}-\mu_o}{S_x/\sqrt{n}}}$, $\sim$ ______________,
   $T=\frac{\overline{X}-\overline{Y}-(\mu_x-\mu_y)}{\hat\sigma_p\sqrt{1/n_x+1/n_y}}\sim$ ______________, where $\hat\sigma=$ ____________________,
   $W=(n_x-1)S_x^2/\sigma^2\sim$ ______________, $F=S_x^2/S_y^2\sim$ ______________,
   key: $\underline{\sigma^2}$, $\underline{\sigma^2}$, $\underline{\perp}$, $\underline{t_{n-1}}$, $\underline{t_{n+m-2}}$, $\underline{\sqrt{\frac{(n-1)S_x^2+(m-1)S_y^2}{n+m-2}}}$, $\underline{\chi^2_{n-1}}$, $\underline{F_{n-1,m-1}}$,
6. $f_{Y_{(j)}}(t)=\binom{n}{j-1,1,n-j}$ _________ **key:** $\underline{(F(t))^{j-1}(f(t))^1(1-F(t))^{n-j}}$.
7. Given two _________ estimators $\hat\theta_1$ and $\hat\theta_2$, eff$(\hat\theta_1,\hat\theta_2)=$ _________, **key:** $\underline{unbiased}$, $\underline{\frac{V(\hat\theta_2)}{V(\hat\theta_1)}}$,
8. $\hat\theta$ is consistent or $\hat\theta\xrightarrow{P}\theta$ if $P(|\hat\theta-\theta|>\epsilon)\to$ _________, _________ >0, **key:** $\underline{0}$, $\underline{\forall\ \epsilon}$,
9. If (1) $E(\hat\theta)=$ ____ and (2) $V(\hat\theta)$ _________, as $n\to\infty$, then $\hat\theta$ is consistent. **key:** $\underline{\theta}$, $\underline{\to 0}$,
10. If $g(\cdot)$ is ______________, $\hat\theta_1\xrightarrow{P}\theta_1$ and $\hat\theta_2\xrightarrow{P}\theta_2$, then $\hat\theta_1\pm\hat\theta_2\xrightarrow{P}$ _________, $\hat\theta_1\hat\theta_2\xrightarrow{P}$ _________, $g(\hat\theta_1)\xrightarrow{P}$ _________, **key:** $\underline{cts}$, $\underline{\theta_1\pm\theta_2}$, $\underline{\theta_1\theta_2}$, $\underline{g(\theta_1)}$,
11. If $X_1,...,X_n$ are i.i.d. from $f(x;\theta)=exp\{$_________ $+g(\theta)+h(x)\}$, $\hat\gamma=G($_________$)$ and _________ $=\gamma(\theta)$, then $\hat\gamma$ is the MVUE of γ. **key:** $\underline{T(x)\psi(\theta)}$, $\underline{\sum_i T(X_i))}$, $\underline{E(\hat\gamma)}$,
12. An MME of θ, is the solution of θ to $\mu_i'(\theta)=$ _________ for k i's, where $\mu_i'(\theta)=$ _________, and k is the dimension of θ. **key:** $\underline{\overline{X^i}}$, $\underline{E(X^i)}$,
13. Given a random sample X_1, ..., X_n from $f(x;\theta)$, their likelihood is $L(\theta)=$ __________, the MLE $\hat\theta$ of θ maximizes _________. If $g(\theta)$ is a _________ function of θ, the MLE of $g(\theta)$ is _________. **key:** $\underline{\prod_i f(X_i;\theta)}$, $\underline{L(\theta)}$. $\underline{1-1}$, $\underline{g(\hat\theta)}$,
14. Given a random sample from $f(x;\theta)$, and a function $g(\theta)$, under certain conditions, $P(g(\hat\theta)\le t)$ ____ $\Phi(\frac{t-g(\theta)}{\hat\sigma_{g(\hat\theta)}})$, where $\hat\theta$ is the ______ of θ, $\hat\sigma_{g(\hat\theta)}=$ ____________________ **key:** $\underline{\approx}$, $\underline{MLE}$, $\underline{\sqrt{\{(\frac{\partial g}{\partial\theta})^2/(-nE(\frac{\partial^2\log f(X;\theta)}{\partial\theta^2}))\}}\Big|_{\theta=\hat\theta}}$,
15. The 5 elements of a test are (1) _________, (2) _________, (3) test statistics (4) _________, (5) _________, **key:** $\underline{H_o}$, $\underline{H_a}$, $\underline{RR}$, $\underline{Conclusion}$,
16. Probability of type I error is _________, Probability of type II error is _________, **key:** $\underline{P(H_a|H_o)}$, $\underline{P(H_o|H_a)}$,
17. For a large sample test for H_o: $\theta=\theta_o$, a test statistic is $Z=$ ______________, a RR is Z_________ if $\theta>\theta_o$; and a RR is __________________ if $\theta\ne\theta_o$; **key:** $\underline{\frac{\hat\theta-\theta_o}{\hat\sigma_{\hat\theta}}}$, $\underline{>z_\alpha}$, $\underline{|Z|>z_{\alpha/2}}$,

18. Sample size for an upper-tail α-level test is $n = (\underline{\hspace{3cm}})^2$ **key:** $\underline{\frac{(z_\alpha+z_\beta)\sigma}{\mu_a-\mu_o}}$,

19. The P-value is $\begin{cases} P(W\underline{\quad}w)|H_o \text{ is correct}) & \text{if } H_a:\ \theta>\theta_o \\ P(W\underline{\quad}w)|H_o \text{ is correct}) & \text{if } H_a:\ \theta<\theta_o \\ \underline{\quad}P(W\underline{\quad}|w|)|H_o \text{ is correct}) & \text{if } H_a:\ \theta\neq\theta_o \end{cases}$ where W is the (Z or T) test statistic and w is the observed value of W. **key:** $\underline{\ge}$, $\underline{\le}$, $\underline{2}$, $\underline{\ge}$,

20. Suppose that $Z \sim N(0,1)$, $X \sim \chi^2(u)$, $Y \sim \chi^2(v)$. If $Z\underline{\quad}X$, $T = \underline{\hspace{2cm}}$, then $T \sim t_u$; If $X\underline{\quad}Y$, $F = \underline{\quad}$, then $F \sim F_{u,v}$ and $X+Y \sim \underline{\hspace{2cm}}$. **key:** $\underline{\perp}$, $\underline{Z/\sqrt{X/u}}$, $\underline{\perp}$, $\underline{\frac{X/u}{Y/v}}$, $\underline{\chi^2(u+v)}$,

21. The MP test for H_o: $\theta=\theta_o$ v.s. H_a: $\theta=\theta_a$, the MP test has the RR satisfying: $\frac{L(\theta_o)}{L(\theta_a)}\underline{\quad}k$ and $P_\theta(RR)=\alpha$ if $\theta = \underline{\quad}$. **key:** $\underline{\le}$, $\underline{\theta_o}$,

22. The Likelihood ratio test for H_o: $\theta\in\Theta_o$ v.s. H_a: $\theta\notin\Theta_o$ has a RR: $\{\lambda\underline{\quad}k\}$, where

$\lambda = \underline{\hspace{2cm}}$; $\hat\theta_o$ is the MLE under $\underline{\quad}$; $\hat\theta$ is the MLE under $\underline{\quad}$;

k satisfies $max\{P(RR):\ \theta\in\Theta_o\} = \underline{\quad}$; if n is large, then $-2\ln\lambda$ is approximated $\underline{\hspace{1.5cm}}$; where $v = \underline{\hspace{2cm}}$; r and $r_o = \#$ of free parameters in Θ and in Θ_o, respectively.

key: $\underline{\le}$, $\underline{\frac{L(\hat\theta_o)}{L(\hat\theta)}}$, $\underline{\Theta_o}$, $\underline{\Theta}$, $\underline{\alpha}$, $\underline{\chi^2(v)}$, $\underline{r-r_o}$

23. Suppose (x_i, Y_i)'s are independent, $\mu_{x_i}(\theta)=E(Y_i)$, the LSE of θ, denoted by $\hat\theta$, minimizes $\underline{\hspace{4cm}}$.

If $\mu_{x_i}(\theta)=\beta_0+\beta_1 x_i$ and $V(Y_i)=\sigma^2$, the LSE $\hat\beta_0 = \underline{\hspace{3cm}}$,

$\hat\beta_1 = \underline{\hspace{3cm}}$, $S_{xy} = \underline{\hspace{3cm}}$, $E(\hat\beta_0) = \underline{\hspace{1cm}}$, $E(\hat\beta_1) = \underline{\hspace{1cm}}$,

$\sigma^2_{\hat\beta_0} = \underline{\hspace{3cm}}$, $\sigma^2_{\hat\beta_1} = \underline{\hspace{3cm}}$, an unbiased estimator of σ^2 is $\hat\sigma^2 = \underline{\hspace{1cm}}$,

key: $\underline{\sum_{i=1}^n [Y_i-\mu_{x_i}(\theta)]^2}$, $\underline{\overline{Y}-\hat\beta_1\overline{x}}$, $\underline{S_{xy}/S_{xx}}$, $\underline{n(\overline{xY}-\overline{x}\overline{Y})}$, $\underline{\beta_0}$, $\underline{\beta_1}$,

$\underline{\sigma^2\overline{x^2}/S_{xx}}$, $\underline{\sigma^2/S_{xx}}$, $\underline{\frac{1}{n-2}(S_{yy}-\hat\beta_1 S_{xy})}$,

If $Y_i \sim \underline{\hspace{4cm}}$, then $\hat\beta_j \sim \underline{\hspace{4cm}}$, $\frac{n-2}{\sigma^2}\hat\sigma^2 \sim \underline{\hspace{1.5cm}}$, $\hat\sigma^2, \hat\beta_0, \hat\beta_1$ are $\underline{\hspace{4cm}}$, $\underline{\hspace{4cm}} \sim t_{n-2}$,

key: $\underline{N(\beta_0+\beta_1 x_i, \sigma^2)}$, $\underline{N(\beta_j, \sigma^2_{\hat\beta_j})}$, $\underline{\chi^2(n-2)}$, $\underline{independent}$, $\underline{\frac{\hat\beta_j-\beta_j}{\hat\sigma_{\hat\beta_j}}}$,

24. Under the Bayes model, conditional on θ, $Y_1, ..., Y_n$ are i.i.d. with $f(y|\theta)$, and $\theta \sim g(\theta)$. The posterior df is $g(\theta|\underline{y}) =$ ____________________, where $\underline{y} = (y_1, ..., y_n)$ the Bayes estimator of $h(\theta)$ is $\hat{h} =$ ______________

key: $\underline{\frac{\prod_{i=1}^n f(y_i|\theta)g(\theta)}{f_{\underline{Y}}(\underline{y})}}$, $\underline{E(h(\theta)|\underline{y})}$,

Formulas for 447:

23. $X \sim \mathcal{G}(\alpha, \beta)$. $f(x) =$ ______________ if $x >$ ____, $\mu =$ _______, $\sigma^2 =$ _______, $\Gamma(\alpha+1) =$ _______, **key:** $\underline{\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}}$, $\underline{0}$, $\underline{\alpha\beta}$, $\underline{\alpha\beta^2}$, $\underline{\alpha\Gamma(\alpha)}$

24. $Exp(\lambda) =$ ______________, $\chi^2(\nu) =$ ______________, **key:** $\underline{\mathcal{G}(1, \lambda)}$, $\underline{\mathcal{G}(\frac{\nu}{2}, 2)}$

25. $X \sim beta(\alpha, \beta)$. $f(x) =$ ______________ if x ________, $\mu =$ _______, where $B(\alpha, \beta) =$ _______,

key: $\underline{\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}}$, $\underline{\in (0, 1)}$, $\underline{\frac{\alpha}{\alpha+\beta}}$, $\underline{\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}}$

41. If Y_i's are i.i.d. with $N(\mu, \sigma^2)$, then $\frac{\overline{Y}-\mu_{\overline{Y}}}{\sigma_{\overline{Y}}} \sim$ _________, $\frac{(n-1)S^2}{\sigma^2} \sim$ _________, S^2 ____ $\overline{Y}$, where $\mu_{\overline{Y}} =$ _________, $\sigma^2_{\overline{Y}} =$ _________ **key:** $\underline{N(0, 1)}$, $\underline{\chi^2(n-1)}$, $\underline{\perp}$, $\underline{\mu}$, $\underline{\sigma^2/n}$

44. If X_1___X_2. **key:** $\underline{\perp}$,

X_i's $\sim$:	$X_1 + X_2 \sim$:	key
$\mathcal{G}(\alpha_i, \beta)$	____________	$\underline{\mathcal{G}(\alpha_1 + \alpha_2, \beta)}$
$\chi^2(v_i)$	____________	$\underline{\chi^2(v_1 + v_2)}$
$Pois(\lambda_i)$	____________	$\underline{Pois(\lambda_1 + \lambda_2)}$
$N(\mu_i, \sigma_i^2)$	____________	$\underline{N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)}$
$bin(n_i, p)$	____________	$\underline{bin(n_1 + n_2, p)}$