**Formulas For Math 447**

**Part A. Fill the blanks of the formulas (30 points)**

1. DeMorgan's laws: $\overline{(A\cap B)} = \underline{\overline{A}\cup\overline{B}}$, and $\overline{(A\cup B)} = \underline{\overline{A}\cap\overline{B}}$.
2. Axioms of probability: (1) $P(A) \geq \underline{0}$, (2) $P(S) = \underline{1}$,
   (3) if $A_i$'s are pairwise $\underline{disjoint}$, then $P(\cup_i A_i) = \sum_i P(A_i)$.
3. If each sample point in the sample space S is equally likely to occur, $P(A) = \frac{N(\_\_\_\_)}{N(\_\_\_\_)}$ **key:** $\underline{A}$, $\underline{S}$
4. If an experiment can be partitioned into 2 steps, with $n_i$ choices in the $i^{th}$ step, then the total # of choices in the experiment is $\underline{n_1 n_2}$
5. $P_r^n = \underline{\frac{n!}{(n-r)!}}$, $C_r^n = \binom{n}{r} = \underline{\frac{n!}{r!(n-r)!}}$,
   $\binom{n}{n_1,n_2,n_3} = \underline{\frac{n!}{n_1!n_2!n_3!}}$, where $n_1+n_2+n_3 = \underline{n}$
6. $P(A\cap B) = \underline{P(A|B)P(B)}$. If $A$ and $B$ are $\underline{independent}$ then $P(A\cap B) = P(A)P(B)$.
7. $P(\ \underline{\overline{A}}\ = 1 - P(A)$. $P(A\cup B) = P(A)+P(B)\ \underline{-P(A\cap B)}$,
   If $A$ and $B$ are $\underline{disjoint}$ then $P(A\cup B) = \underline{P(A)+P(B)}$.
8. Bayes' Rule: If $\{B_1,...,B_k\}$ is a partition of $S$ and $P(A), P(B_j) > 0$, then
   $P(B_j|A) = \underline{\frac{P(A|B_j)P(B_j)}{\sum_{k=1}^k P(A|B_k)P(B_k)}}$
9. $X \sim bin(n,p)$: $f(i) = \underline{\binom{n}{i}p^i q^{n-i}}$, if $i \in \{0,...,n\}$, $\mu = \underline{np}$, $\sigma^2 = \underline{npq}$ where $q = 1-p$
10. $X \sim G(p)$. $f(i) = q^{i-1}p$ if $i \geq \underline{1}$, $\mu = \underline{1/p}$, $\sigma^2 = \underline{q/p^2}$
11. $X \sim Nbin(r,p)$, $f(i) = \binom{i-1}{r-1}p^r q^{i-r}$ if $i \geq \underline{r}$, $\mu = \underline{r/p}$, $\sigma^2 = \underline{rq/p^2}$
12. $X \sim HG(N,n,r)$. $f(i) = \underline{\frac{\binom{r}{i}\binom{N-r}{n-i}}{\binom{N}{n}}}$, $\mu = \underline{nr/N}$, $\underline{\sigma^2} = \frac{nr(N-r)(N-n)}{N^2(N-1)}$.
13. $X \sim Pois(\lambda)$. $f(i) = \underline{\frac{e^{-\lambda}\lambda^i}{i!}}$, if $i \geq 0$. $\mu = \underline{\lambda}$, $\sigma^2 = \underline{\lambda}$
14. Tchebysheff's Inequality. $P(|X-\mu| > k\sigma)\ \underline{\leq 1/k^2}$
15. $Y = g(X)$. $E(g(X)) = \begin{cases} \sum_y y f_Y(y) & \text{dis} \\ \int y f_Y(y)dy & \text{cts} \end{cases} = \begin{cases} \underline{\sum_x g(x)f_X(x)} & \text{dis} \\ \underline{\int g(x)f_X(x)dx} & \text{cts} \end{cases}$, $\mu_Y = \underline{E(Y)}$,
    $\sigma_Y^2 = \underline{E(Y^2) - \mu_Y^2}$
16. The mgf of $X$ is $M(t) = \underline{E(e^{Xt})}$, $\frac{d^k M(t)}{dt^k}\Big|_{t=0} = \underline{E(X^k)}$
17. A cdf $F(t)$ ($= \underline{P(X \leq t)}$, ) satisfying (1) $F(-\infty) = \underline{0}$, and $F(\infty) = \underline{1}$, (2) $F(x+) = \underline{F(x)}$,
    (3) $F(x)\ \underline{\uparrow}$, . Moreover, $F(b)-F(a) = \mathrm{P}(\underline{a < X \leq b})$
18. $F(t) = \begin{cases} \sum_{x\leq t} \underline{f(x)} & \text{dis} \\ \int_{-\infty}^t \underline{f(x)dx} & \text{cts} \end{cases}$, $f(t) = \begin{cases} \underline{F(t)-F(t-)} & \text{dis} \\ \underline{F'(t)} & \text{cts} \end{cases}$
19. For discrete r.v. $X$, the p.f. $f$ satisfies $f(t) = P(\ \underline{X = t})$, $f(t) \in \underline{[0,1]}$, and $\sum_t f(t) = \underline{1}$,
    For cts r.v. $X$, the d.f. $f$ satisfies $f(t)\ \underline{\geq 0}$, and $\int f(t)dx = \underline{1}$
20. $E(aX+b) = \underline{aE(X)+b}$, $Var(aX+b) = \underline{a^2\sigma_X^2}$
21. $X \sim U(a,b)$. $f(x) = \underline{\frac{1}{b-a}}$, if $x \in (a,b)$. $\mu = \underline{\frac{a+b}{2}}$, $\sigma^2 = \underline{\frac{(b-a)^2}{12}}$
22. $X \sim N(\mu,\sigma^2)$. $f(x) = \underline{\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$, $\frac{X-\mu}{\sigma} \sim \underline{N(0,1)}$
23. $X \sim \mathcal{G}(\alpha,\beta)$. $f(x) = \underline{\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}}$, if $x > \underline{0}$, $\mu = \underline{\alpha\beta}$, $\sigma^2 = \underline{\alpha\beta^2}$, $\Gamma(\alpha+1) = \underline{\alpha\Gamma(\alpha)}$
24. $Exp(\lambda) = \underline{\mathcal{G}(1,\lambda)}$, $\chi^2(\nu) = \underline{\mathcal{G}(\frac{\nu}{2},2)}$

25. $X \sim beta(\alpha,\beta)$. $f(x) = \underline{\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}}$, if $x \in \underline{(0,1)}$, $\mu = \underline{\frac{\alpha}{\alpha+\beta}}$, where $B(\alpha,\beta) = \underline{\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}}$

26. The cdf of $(X,Y)$ is $F(x,y) = P(\underline{X \le x, Y \le y}) = \begin{cases} \underline{\int_{-\infty}^{x}\int_{-\infty}^{y} f(t,s)dsdt} & \text{cts} \\ \underline{\sum_{t\le x}\sum_{s\le y} f(t,s)} & \text{dis} \end{cases}$

27. For continuous $(X,Y)$, their d.f. $f(x,y) = \underline{\frac{\partial^2 F(x,y)}{\partial x \partial y}}$, $f(x,y) \ge \underline{0}$, $\int\int f(x,y)dxdy = \underline{1}$,
For discrete $(X,Y)$, their d.f. $f(x,y) = \underline{P\{X=x, Y=y\}}$, $f(x,y) \in \underline{[0,1]}$, $\sum_x\sum_y f(x,y) = \underline{1}$

28. $P((X,Y)\in A) = \begin{cases} \underline{\int\int_{(x,y)\in A} f(x,y)dxdy} & \text{cts} \\ \underline{\sum_{(x,y)\in A} f(x,y)} & \text{dis} \end{cases}$

29. $f_X(x) = \begin{cases} \int f(x,y)\underline{dy} & \text{cts} \\ \underline{\sum_y} f(x,y) & \text{dis} \end{cases}$

30. $f_{X|Y}(x|y) = \underline{\frac{f(x,y)}{f_Y(y)}}$, $F_{X|Y}(x|y) = P(\ \underline{(X \le x|Y=y)}$

31. $E(g(X,Y)) = \begin{cases} \underline{\int\int g(x,y)f(x,y)dxdy} & \text{cts} \\ \underline{\sum_x\sum_y g(x,y)f(x,y)} & \text{dis} \end{cases}$

32. The cts or discrete $X$ and $Y$ are independent iff $F(x,y) = \underline{F_X(x)F_Y(y)}$ iff $f(x,y) = \underline{f_X(x)f_Y(y)}$

33. $E(c) = \underline{c}$, $E(ag(X,Y)+bh(X,Y)) = \underline{aE(g(X,Y))+bE(h(X,Y))}$

34. $Cov(X,Y) = \underline{E(XY)-E(X)E(Y)}$, $V(aX+bY) = \underline{a^2V(X)+b^2V(Y)+2abCov(X,Y)}$ , $\rho = \underline{\frac{Cov(X,Y)}{\sigma_X,\sigma_Y}}$,

35. $(Y_1,Y_2,...,Y_k) \sim M(n,p_1,...,p_k)$. $f(y_1,...,y_k) = \underline{\binom{n}{y_1,...,y_k}}$, $p_1^{y_1}\cdots p_k^{y_k}$, $y_i \in \{0,1,...,n\}$, $\sum_i p_i = \underline{1}$, and $\sum_i y_i = \underline{n}$, $Cov(Y_i,Y_j) = \underline{-np_ip_j}$, if $i \ne j$, $Y_i \sim \underline{bin(n,p_i)}$

36. $E(X|Y=y) = \begin{cases} \underline{\int xf(x|y)dx} & \text{cts} \\ \underline{\sum_x xf(x|y)} & \text{dis} \end{cases}$ . $E(E(X|Y)) = \underline{E(X)}$,
$E(V(X|Y)) + V(E(X|Y)) = \underline{V(X)}$

37. $U = h(Y_1,...,Y_n)$, where $Y_i$'s are continuous. $F_U(t) = \int\cdots\int_A f(y_1,...,y_n)dy_1\cdots dy_n$, where $A = \underline{\{h(y_1,...,y_n)\le t\}}$, $f_U(t) = \underline{F_U'(t)}$

38. $U = h(Y)$, where $h$ is $\underline{monotone}$, and $Y$ is cts. $f_U(u) = f_Y(\ \underline{h^{-1}(u)}\ )\cdot \underline{\left|\frac{\partial h^{-1}}{\partial u}\right|}$

39. If $X$ and $Y$ are independent, then $Cov(X,Y) = \underline{0}$, $E(g(X)h(Y)) = \underline{E(g(X))E(h(Y))}$ $M_{X+Y}(t) = \underline{M_X(t)\cdot M_Y(t)}$
**Hereafter, let $Y_1$, ..., $Y_n$ be a random sample of $Y$.**

40. Let $Y_1$, ..., $Y_n$ be a random sample of $Y$. $\overline{Y} = \underline{\frac{1}{n}\sum_{i=1}^n Y_i}$, $S^2 = S_Y^2 = \underline{\frac{1}{n-1}\sum_{i=1}^n (Y_i-\overline{Y})^2}$

41. If $Y \sim N(\mu,\sigma^2)$, $\frac{\overline{Y}-\mu_{\overline{Y}}}{\sigma_{\overline{Y}}} \sim \underline{N(0,1)}$, $\frac{(n-1)S^2}{\sigma^2} \sim \underline{\chi^2(n-1)}$, $\sqrt{n}\frac{\overline{Y}-\mu}{S} \sim \underline{t_{n-1}}$, where $\mu_{\overline{Y}} = \underline{\mu}$, $\sigma^2_{\overline{Y}} = \underline{\sigma^2/n}$

42. $F_{\overline{Y}}(t) \approxeq, \Phi(\ \underline{\frac{t-\mu_{\overline{Y}}}{\sigma_{\overline{Y}}}},\ )$, where $\Phi(t)$ is the cdf of $\underline{N(0,1)}$

## Review formulas for Math 448,

1. Estimator of $\mu$ is $\overline{X}$ where $\overline{X}$ = ______________, Estimator of $\sigma^2$ is $S^2$, where $S^2$ = ______________, **key:** $\underline{\sum_i X_i/n}$, $\underline{\frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X})^2}$.
2. An estimator $\hat{\theta}$ is unbiased if ______________, bias $B(\hat{\theta})$ = ______________, MSE = __________________, (**key:** $\underline{E(\hat{\theta}) = \theta}$, $\underline{E(\hat{\theta}) - \theta}$, $\underline{V(\hat{\theta}) + (B(\hat{\theta}))^2}$,
3. $100(1-\alpha)\%$ CI for $\theta$: ________________ = _____ **key:** $\underline{P(a \le \theta \le b)}$, $\underline{1-\alpha}$.
4. $100(1-\alpha)\%$ large sample CI for $\theta$: ______________ **key:** $\underline{\hat{\theta} \pm z_{\alpha/2}\hat{\sigma}_{\hat{\theta}}}$.
5. If (1) $X_1$, ..., $X_n$ are i.i.d. from $N(\mu_x, \_\_\_)$, (2) $Y_1$, ..., $Y_m$ are i.i.d. from $N(\mu_y, \_\_\_)$, and (3) $X_i$'s ___ $Y_j$'s, then $T = \frac{\overline{X}-\mu_o}{S_x/\sqrt{n}}$, $\sim$ ______________,
   $T = \frac{\overline{X}-\overline{Y}-(\mu_x-\mu_y)}{\hat{\sigma}_p\sqrt{1/n+1/m}} \sim$ ______________, where $\hat{\sigma}$ = ____________________,
   $W = (n-1)S_x^2/\sigma^2 \sim$ ______________, $F = S_x^2/S_y^2 \sim$ ______________,
   **key:** $\underline{\sigma^2}$, $\underline{\sigma^2}$, $\underline{\perp}$, $\underline{t_{n-1}}$, $\underline{t_{n+m-2}}$, $\underline{\sqrt{\frac{(n-1)S_x^2+(m-1)S_y^2}{n+m-2}}}$, $\underline{\chi^2_{n-1}}$, $\underline{F_{n-1,m-1}}$,
6. $f_{Y_{(j)}}(t) = \binom{n}{j-1,1,n-j}$ ________ **key:** $\underline{(F(t))^{j-1}(f(t))^1(1-F(t))^{n-j}}$.
7. Given two ________ estimators $\hat{\theta}_1$ and $\hat{\theta}_2$, eff$(\hat{\theta}_1, \hat{\theta}_2)$ = ________,
   **key:** *unbiased*, $\underline{\frac{V(\hat{\theta}_2)}{V(\hat{\theta}_1)}}$,
8. $\hat{\theta}$ is consistent or $\hat{\theta} \xrightarrow{P} \theta$ if $P(|\hat{\theta} - \theta| > \epsilon) \to$ ________, ________ $> 0$, **key:** $\underline{0}$, $\underline{\forall\ \epsilon}$,
9. If (1) $E(\hat{\theta})$ = ____ and (2) $V(\hat{\theta})$ ________, as $n \to \infty$, then $\hat{\theta}$ is consistent.
   **key:** $\underline{\theta}$, $\underline{\to 0}$,
10. If $g(\cdot)$ is ____________, $\hat{\theta}_1 \xrightarrow{P} \theta_1$ and $\hat{\theta}_2 \xrightarrow{P} \theta_2$,
    then $\hat{\theta}_1 \pm \hat{\theta}_2 \xrightarrow{P}$ ________, $\hat{\theta}_1\hat{\theta}_2 \xrightarrow{P}$ ________, $g(\hat{\theta}_1) \xrightarrow{P}$ ________,
    **key:** *cts*, $\underline{\theta_1 \pm \theta_2}$, $\underline{\theta_1\theta_2}$, $\underline{g(\theta_1)}$,
11. If $X_1, ..., X_n$ are i.i.d. from $f(x;\theta) = exp\{$________ $+ g(\theta) + h(x)\}$, $\hat{\gamma} = G($________$)$ and ________ $= \gamma(\theta)$, then $\hat{\gamma}$ is the MVUE of $\gamma$.
    **key:** $\underline{T(x)\psi(\theta)}$, $\underline{\sum_i T(X_i))}$, $\underline{E(\hat{\gamma})}$,
12. An MME of $\theta$, is the solution of $\theta$ to $\mu_i'(\theta)$ = ________ for $k$ $i$'s, where $\mu_i'(\theta)$ = ________, and $k$ is the dimension of $\theta$. **key:** $\underline{\overline{X^i}}$, $\underline{E(X^i)}$,
13. Given a random sample $X_1$, ..., $X_n$ from $f(x;\theta)$, their likelihood is $L(\theta)$ = __________, the MLE $\hat{\theta}$ of $\theta$ maximizes ________. If $g(\theta)$ is a ________ function of $\theta$, the MLE of $g(\theta)$ is ________.
    **key:** $\underline{\prod_i f(X_i;\theta)}$, $\underline{L(\theta)}$. $\underline{1-1}$, $\underline{g(\hat{\theta})}$,
14. Given a random sample from $f(x;\theta)$, and a function $g(\theta)$, under certain conditions, $P(g(\hat{\theta}) \le t)$ ____ $\Phi(\frac{t-g(\theta)}{\hat{\sigma}_{g(\hat{\theta})}})$, where $\hat{\theta}$ is the _____ of $\theta$, $\hat{\sigma}_{g(\hat{\theta})}$ = __________________
    **key:** $\approx$, *MLE*, $\underline{\sqrt{\{(\frac{\partial g}{\partial \theta})^2/(-nE(\frac{\partial^2 \log f(X;\theta)}{\partial \theta^2}))\}|_{\theta=\hat{\theta}}}}$,
15. The 5 elements of a test are (1) ________, (2) ________, (3) test statistics (4) ________, (5) ________, **key:** $\underline{H_o}$, $\underline{H_a}$, *RR*, *Conclusion*,
16. Probability of type I error is ________, Probability of type II error is ________, **key:** $\underline{P(H_a|H_o)}$, $\underline{P(H_o|H_a)}$,
17. For a large sample test for $H_o$: $\theta = \theta_o$, a test statistic is $Z$ = ______________, a RR is $Z$________ if $\theta > \theta_o$; and a RR is ________________ if $\theta \neq \theta_o$; **key:** $\underline{\frac{\hat{\theta}-\theta_o}{\hat{\sigma}_{\hat{\theta}}}}$, $\underline{> z_\alpha}$, $\underline{|Z| > z_{\alpha/2}}$,

18. Sample size for an upper-tail $\alpha$-level test is $n = (\underline{\qquad\qquad\qquad})^2$ **key:** $\underline{\frac{(z_\alpha+z_\beta)\sigma}{\mu_a-\mu_o}}$,

19. The P-value is $\begin{cases} P(W\underline{\quad}w)|H_o \text{ is correct}) & \text{if } H_a: \theta > \theta_o \\ P(W\underline{\quad}w)|H_o \text{ is correct}) & \text{if } H_a: \theta < \theta_o \\ \underline{\quad}P(W\underline{\quad}|w|)|H_o \text{ is correct}) & \text{if } H_a: \theta \neq \theta_o \end{cases}$ where $W$ is the ($Z$ of $T$) test statistic and $w$ is the observed value of $W$. **key:** $\underline{\geq}$, $\underline{\leq}$, $\underline{2}$, $\underline{\geq}$,

20. Suppose that $Z \sim N(0,1)$, $X \sim \chi^2(u)$, $Y \sim \chi^2(v)$. If $Z\underline{\quad}X$, $T = \underline{\qquad\qquad}$, then $T \sim t_v$; If $X\underline{\quad}Y$, $F = \underline{\quad}$, then $F \sim F_{u,v}$ and $X + Y \sim \underline{\qquad\qquad}$. **key:** $\underline{\perp}$, $\underline{T = Z/\sqrt{X/u}}$, $\underline{\perp}$, $\underline{\frac{X/u}{Y/v}}$, $\underline{\chi^2(u+v)}$,

21. The MP test for $H_o$: $\theta = \theta_o$ v.s. $H_a$: $\theta = \theta_a$, the MP test has the RR satisfying: $\frac{L(\theta_o)}{L(\theta_a)}\underline{\quad}k$ and $P_\theta(RR) = \alpha$ if $\theta = \underline{\quad}$. **key:** $\underline{\leq}$, $\underline{\theta_o}$,

22. The Likelihood ratio test for $H_o$: $\theta \in \Theta_o$ v.s. $H_a$: $\theta \notin \Theta_o$ has a RR: $\{\lambda\underline{\quad}k\}$, where

$\lambda = \underline{\qquad\qquad}$; $\hat{\theta}_o$ is the MLE under $\underline{\quad}$; $\hat{\theta}$ is the MLE under $\underline{\quad}$; $k$ satisfies $max\{P(RR): \theta \in \Theta_o\} = \underline{\quad}$; if $n$ is large, then $-2\ln\lambda$ is approximated $\underline{\qquad}$; where $v = \underline{\qquad\qquad}$; $r$ and $r_o$ = # of free parameters in $\Theta$ and in $\Theta_o$, respectively.

**key:** $\underline{\leq}$, $\underline{\frac{L(\hat{\theta}_o)}{L(\hat{\theta})}}$, $\underline{\Theta_o}$, $\underline{\Theta}$, $\underline{\alpha}$, $\underline{\chi^2(v)}$, $\underline{r - r_o}$

23. Suppose $(x_i, Y_i)$'s are independent, $\mu_{x_i}(\theta) = E(Y_i)$, the LSE of $\theta$, denoted by $\hat{\theta}$, minimizes $\underline{\qquad\qquad\qquad}$.

If $\mu_{x_i}(\theta) = \beta_0 + \beta_1 x_i$ and $V(Y_i) = \sigma^2$, the LSE $\hat{\beta}_0 = \underline{\qquad\qquad\qquad}$,

$\hat{\beta}_1 = \underline{\qquad\qquad\qquad}$, $S_{xy} = \underline{\qquad\qquad\qquad}$, $E(\hat{\beta}_0) = \underline{\quad}$, $E(\hat{\beta}_1) = \underline{\quad}$,

$\sigma^2_{\hat{\beta}_0} = \underline{\qquad\qquad\qquad}$, $\sigma^2_{\hat{\beta}_1} = \underline{\qquad\qquad\qquad}$, an unbiased estimator of $\sigma^2$ is $\hat{\sigma}^2$=$\underline{\quad}$,

**key:** $\underline{\sum_{i=1}^n [Y_i - \mu_{x_i}(\theta)]^2}$, $\underline{\overline{Y} - \hat{\beta}_1\overline{x}}$, $\underline{S_{xy}/S_{xx}}$, $\underline{n(\overline{xY} - \overline{x}\overline{Y})}$, $\underline{\beta_0}$, $\underline{\beta_1}$, $\underline{\sigma^2\overline{x^2}/S_{xx}}$, $\underline{\sigma^2/S_{xx}}$, $\underline{\frac{1}{n-2}(S_{yy} - \hat{\beta}_1 S_{xy})}$,

If $Y_i \sim \underline{\qquad\qquad\qquad}$, then $\hat{\beta}_j \sim \underline{\qquad\qquad\qquad}$, $\frac{n-2}{\sigma^2}\hat{\sigma}^2 \sim \underline{\quad}$, $\hat{\sigma}^2, \hat{\beta}_0, \hat{\beta}_1$ are $\underline{\qquad\qquad\qquad}$, $\underline{\qquad\qquad\qquad} \sim t_{n-2}$,

**key:** $\underline{N(\beta_0 + \beta_1 x_i, \sigma^2)}$, $\underline{N(\beta_j, \sigma^2_{\hat{\beta}_j})}$, $\underline{\chi^2(n-2)}$, $\underline{independent}$, $\underline{\frac{\hat{\beta}_j - \beta_j}{\hat{\sigma}_{\hat{\beta}_j}}}$,

24. Under the Bayes model, conditional on $\theta$, $Y_1$, ..., $Y_n$ are i.i.d. with $f(y|\theta)$, and $\theta \sim g(\theta)$. The posterior df is $g(\theta|\underline{y}) =$ ____________________, where $\underline{y} = (y_1, ..., y_n)$ the Bayes estimator of $h(\theta)$ is $\hat{h} =$ ______________

**key:** $\underline{\frac{\prod_{i=1}^n f(y_i|\theta)g(\theta)}{f_{\underline{Y}}(\underline{y})}}$, $\underline{E(h(\theta)|\underline{y})}$,

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23. $X \sim \mathcal{G}(\alpha, \beta)$. $f(x) =$ _______________ if $x >$ _____, $\mu =$ ________, $\sigma^2 =$ ________, $\Gamma(\alpha+1) =$ ________, **key:** $\underline{\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}}$, $\underline{0}$, $\underline{\alpha\beta}$, $\underline{\alpha\beta^2}$, $\underline{\alpha\Gamma(\alpha)}$

24. $Exp(\lambda) =$ _______________, $\chi^2(\nu) =$ _______________, **key:** $\underline{G(1, \lambda)}$, $\underline{G(\frac{\nu}{2}, 2)}$

25. $X \sim beta(\alpha, \beta)$. $f(x) =$ _______________ if $x$ _________, $\mu =$ ________, where $B(\alpha, \beta) =$ ________,

**key:** $\underline{\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}}$, $\underline{\in (0, 1)}$, $\underline{\frac{\alpha}{\alpha+\beta}}$, $\underline{\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}}$

41. If $Y_i$'s are i.i.d. with $N(\mu, \sigma^2)$, then $\frac{\overline{Y}-\mu_{\overline{Y}}}{\sigma_{\overline{Y}}} \sim$ _________, $\frac{(n-1)S^2}{\sigma^2} \sim$ _________, $S^2$ _____ $\overline{Y}$, where $\mu_{\overline{Y}} =$ _________, $\sigma^2_{\overline{Y}} =$ _________ **key:** $\underline{N(0, 1)}$, $\underline{\chi^2(n-1)}$, $\underline{\perp}$, $\underline{\mu}$, $\underline{\sigma^2/n}$

44. If $X_1$___$X_2$. **key:** $\underline{\perp}$,

| $X_i$'s $\sim$: | $X_1 + X_2 \sim$: | key |
|---|---|---|
| $G(\alpha_i, \beta)$ | ____________ | $\underline{G(\alpha_1 + \alpha_2, \beta)}$ |
| $\chi^2(v_i)$ | ____________ | $\underline{\chi^2(v_1 + v_2)}$ |
| $Pois(\lambda_i)$ | ____________ | $\underline{Pois(\lambda_1 + \lambda_2)}$ |
| $N(\mu_i, \sigma_i^2)$ | ____________ | $\underline{N(\mu_x + \mu_y, \sigma_1^2 + \sigma_2^2)}$ |
| $bin(n_i, p)$ | ____________ | $\underline{bin(n_1 + n_2, p)}$ |