Formulas For Math 447

Part A. Fill the blanks of the formulas (30 points)

1. DeMorgan's laws: $\overline{(A \cap B)} = \underline{\overline{A} \cup \overline{B}}$, and $\overline{(A \cup B)} = \underline{\overline{A} \cap \overline{B}}$.
2. Axioms of probability: (1) $P(A) \geq \underline{0}$, (2) $P(S) = \underline{1}$,
   (3) if A_i's are pairwise $\underline{disjoint}$, then $P(\cup_i A_i) = \sum_i P(A_i)$.
3. If each sample point in the sample space S is equally likely to occur, $P(A) = \frac{N(\_\_\_\_)}{N(\_\_\_\_)}$ **key:** $\underline{A}$, $\underline{S}$
6. $P(A \cap B) = \underline{P(A|B)P(B)}$. If A and B are $\underline{independent}$ then $P(A \cap B) = P(A)P(B)$.
7. $P(\ \underline{\overline{A}}\ ) = 1 - P(A)$. $P(A \cup B) = P(A) + P(B)\ \underline{-P(A \cap B)}$,
   If A and B are $\underline{disjoint}$ then $P(A \cup B) = P(A) + P(B)$.
9. $X \sim bin(n,p)$: $f(i) = \underline{\binom{n}{i} p^i q^{n-i}}$, if $i \in \{0, ..., n\}$, $\mu = \underline{np}$, $\sigma^2 = \underline{npq}$ where $q = 1 - p$
13. $X \sim Pois(\lambda)$. $f(i) = \underline{\frac{e^{-\lambda}\lambda^i}{i!}}$, if $i \geq 0$. $\mu = \underline{\lambda}$, $\sigma^2 = \underline{\lambda}$
14. Tchebysheff's Inequality. $P(|X - \mu| > k\sigma)\ \underline{\leq 1/k^2}$
15. $Y = g(X)$. $E(g(X)) = \begin{cases} \sum_y y f_Y(y) & \text{dis} \\ \int y f_Y(y)dy & \text{cts} \end{cases} = \begin{cases} \underline{\sum_x g(x) f_X(x)} & \text{dis} \\ \underline{\int g(x) f_X(x)dx} & \text{cts} \end{cases}$, $\mu_Y = \underline{E(Y)}$,
    $\sigma_Y^2 = \underline{E(Y^2) - \mu_Y^2}$
17. A cdf $F(t)$ $(= \underline{P(X \leq t)},\ )$ satisfying (1) $F(-\infty) = \underline{0}$, and $F(\infty) = \underline{1}$, (2) $F(x+) = \underline{F(x)}$,
    (3) $F(x)\ \underline{\uparrow}$. Moreover, $F(b) - F(a) = \text{P}(\underline{a < X \leq b})$
18. $F(t) = \begin{cases} \sum_{x \leq t} \underline{f(x)} & \text{dis} \\ \int_{-\infty}^{t} \underline{f(x)dx} & \text{cts} \end{cases}$, $f(t) = \begin{cases} \underline{F(t) - F(t-)} & \text{dis} \\ \underline{F'(t)} & \text{cts} \end{cases}$
19. For discrete r.v. X, the p.f. f satisfies $f(t) = P(\ \underline{X = t})$, $f(t) \in \underline{[0,1]}$, and $\sum_t f(t) = \underline{1}$,
    For cts r.v. X, the d.f. f satisfies $f(t)\ \underline{\geq 0}$, and $\int f(t)dx = \underline{1}$
20. $E(aX + b) = \underline{aE(X) + b}$, $Var(aX + b) = \underline{a^2\sigma_X^2}$
21. $X \sim U(a,b)$. $f(x) = \underline{\frac{1}{b-a}}$, if $x \in (a,b)$. $\mu = \underline{\frac{a+b}{2}}$, $\sigma^2 = \underline{\frac{(b-a)^2}{12}}$
22. $X \sim N(\mu, \sigma^2)$. $f(x) = \underline{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$, $\frac{X-\mu}{\sigma} \sim \underline{N(0,1)}$
23. $X \sim \mathcal{G}(\alpha, \beta)$. $f(x) = \underline{\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}}$, if $x > \underline{0}$, $\mu = \underline{\alpha\beta}$, $\sigma^2 = \underline{\alpha\beta^2}$, $\Gamma(\alpha + 1) = \underline{\alpha\Gamma(\alpha)}$
24. $Exp(\lambda) = \underline{\mathcal{G}(1, \lambda)}$, $\chi^2(\nu) = \underline{\mathcal{G}(\frac{\nu}{2}, 2)}$
26. The cdf of (X,Y) is $F(x,y) = P(\underline{X \leq x, Y \leq y}) = \begin{cases} \underline{\int_{-\infty}^{x} \int_{-\infty}^{y} f(t,s)dsdt} & \text{cts} \\ \underline{\sum_{t \leq x} \sum_{s \leq y} f(t,s)} & \text{dis} \end{cases}$
27. For continuous (X,Y), their d.f. $f(x,y) = \underline{\frac{\partial^2 F(x,y)}{\partial x \partial y}}$, $f(x,y) \geq \underline{0}$, $\int\int f(x,y)dxdy = \underline{1}$,
    For discrete (X,Y), their d.f. $f(x,y) = \underline{P\{X = x, Y = y\}}$, $f(x,y) \in \underline{[0,1]}$,
    $\sum_x \sum_y f(x,y) = \underline{1}$
28. $P((X,Y) \in A) = \begin{cases} \underline{\int\int_{(x,y)\in A} f(x,y)dxdy} & \text{cts} \\ \underline{\sum_{(x,y)\in A} f(x,y)} & \text{dis} \end{cases}$

29. $f_X(x) = \begin{cases} \underline{\int f(x,y)dy} & \text{cts} \\ \underline{\sum_y f(x,y)} & \text{dis} \end{cases}$

30. $f_{X|Y}(x|y) = \underline{\frac{f(x,y)}{f_Y(y)}}$, $F_{X|Y}(x|y) = P\ (\underline{X \leq x|Y = y})$

31. $E(g(X,Y)) = \begin{cases} \underline{\int\int g(x,y)f(x,y)dxdy} & \text{cts} \\ \underline{\sum_x \sum_y g(x,y)f(x,y)} & \text{dis} \end{cases}$

32. The cts or discrete X and Y are independent iff $F(x,y) = \underline{F_X(x)F_Y(y)}$ iff $f(x,y) = \underline{f_X(x)f_Y(y)}$

33. $E(c) = \underline{c}$, $E(ag(X,Y) + bh(X,Y)) = \underline{aE(g(X,Y)) + bE(h(X,Y))}$

34. $Cov(X,Y) = \underline{E(XY) - E(X)E(Y)}$, $V(aX + bY) = \underline{a^2V(X) + b^2V(Y) + 2abCov(X,Y)}$, $\rho = \underline{\frac{Cov(X,Y)}{\sigma_X \sigma_Y}}$,

35. $(Y_1, Y_2, ..., Y_k) \sim M(n, p_1, ..., p_k)$. $f(y_1, ..., y_k) = \underline{\binom{n}{y_1,...,y_k}}\ p_1^{y_1} \cdots p_k^{y_k}$, $y_i \in \{0, 1, ..., n\}$, $\sum_i p_i = \underline{1}$, and $\sum_i y_i = \underline{n}$, $Cov(Y_i, Y_j) = \underline{-np_ip_j}$, if $i \neq j$, $Y_i \sim \underline{bin(n, p_i)}$

36. $E(X|Y = y) = \begin{cases} \underline{\int xf(x|y)dx} & \text{cts} \\ \underline{\sum_x xf(x|y)} & \text{dis} \end{cases}$. $E(E(X|Y)) = \underline{E(X)}$,
$E(V(X|Y)) + V(E(X|Y)) = \underline{V(X)}$

37. $U = h(Y_1, ..., Y_n)$, where Y_i's are continuous. $F_U(t) = \int \cdots \int_A f(y_1, ..., y_n)dy_1 \cdots dy_n$, where $A = \underline{\{h(y_1, ..., y_n) \leq t\}}$, $f_U(t) = \underline{F'_U(t)}$

38. $U = h(Y)$, where h is <u>monotone</u>, and Y is cts. $f_U(u) = f_Y(\ \underline{h^{-1}(u)}\ ) \cdot \underline{\left|\frac{\partial h^{-1}}{\partial u}\right|}$

39. If X and Y are independent, then $Cov(X,Y) = \underline{0}$, $E(g(X)h(Y)) = \underline{E(g(X))E(h(Y))}$
$M_{X+Y}(t) = \underline{M_X(t) \cdot M_Y(t)}$

Hereafter, let Y_1, ..., Y_n be a random sample of Y.

40. Let Y_1, ..., Y_n be a random sample of Y. $\overline{Y} = \underline{\frac{1}{n}\sum_{i=1}^n Y_i}$, $S^2 = S_Y^2 = \underline{\frac{1}{n-1}\sum_{i=1}^n (Y_i - \overline{Y})^2}$

41. If $Y \sim N(\mu, \sigma^2)$, $\frac{\overline{Y} - \mu_{\overline{Y}}}{\sigma_{\overline{Y}}} \sim \underline{N(0,1)}$, $\frac{(n-1)S^2}{\sigma^2} \sim \underline{\chi^2(n-1)}$, $\sqrt{n}\frac{\overline{Y} - \mu}{S} \sim \underline{t_{n-1}}$, where $\mu_{\overline{Y}} = \underline{\mu}$, $\sigma^2_{\overline{Y}} = \underline{\sigma^2/n}$

42. $F_{\overline{Y}}(t) \approx \Phi(\ \underline{\frac{t - \mu_{\overline{Y}}}{\sigma_{\overline{Y}}}}\ )$, where $\Phi(t)$ is the cdf of $\underline{N(0,1)}$

Review formulas for Math 448,

1. Estimator of μ is $\overline{X}$ where $\overline{X} =$ ______________, Estimator of σ^2 is S^2, where $S^2 =$ ______________, **key:** $\underline{\sum_i X_i/n}$, $\underline{\frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X})^2}$.

2. An estimator $\hat{\theta}$ is unbiased if ______________, bias $B(\hat{\theta}) =$ ______________, MSE = ________________, (**key:** $\underline{E(\hat{\theta}) = \theta}$, $\underline{E(\hat{\theta}) - \theta}$, $\underline{V(\hat{\theta}) + (B(\hat{\theta}))^2}$,

3. $100(1-\alpha)\%$ CI for θ: ________________ = ______ **key:** $\underline{P(a \leq \theta \leq b)}$, $\underline{1 - \alpha}$.

4. $100(1-\alpha)\%$ large sample CI for θ: ______________ **key:** $\underline{\hat{\theta} \pm z_{\alpha/2}\hat{\sigma}_{\hat{\theta}}}$.

5. If (1) X_1, ..., X_n are i.i.d. from $N(\mu_1,$ ___$)$, (2) Y_1, ..., Y_m are i.i.d. from $N(\mu_2,$ ___$)$, and (3) X_i's ___ Y_j's, then $T = \underline{\frac{\overline{X} - \mu_o}{S_x/\sqrt{n}}}$, $\sim$ ______________,
$T = \underline{\frac{\overline{X} - \overline{Y} - (\mu_x - \mu_y)}{\hat{\sigma}_p\sqrt{1/n_x + 1/n_y}}} \sim$ ______________, where $\hat{\sigma} =$ ____________________,
$W = (n_x - 1)S_x^2/\sigma^2 \sim$ ______________, $F = S_x^2/S_y^2 \sim$ ______________,

key: $\underline{\sigma^2}$, $\underline{\sigma^2}$, $\underline{\perp}$, $\underline{t_{n-1}}$, $\underline{t_{n+m-2}}$, $\underline{\sqrt{\frac{(n-1)S_x^2+(m-1)S_y^2}{n+m-2}}}$, $\underline{\chi^2_{n-1}}$, $\underline{F_{n-1,m-1}}$,

6. $f_{Y_{(j)}}(t) = \binom{n}{j-1,1,n-j}$ ________ **key:** $\underline{(F(t))^{j-1}(f(t))^1(1-F(t))^{n-j}}$.
7. Given two ________ estimators $\hat\theta_1$ and $\hat\theta_2$, eff$(\hat\theta_1, \hat\theta_2) =$ ________,
   key: *unbiased*, $\underline{\frac{V(\hat\theta_2)}{V(\hat\theta_1)}}$,
8. $\hat\theta$ is consistent or $\hat\theta \xrightarrow{P} \theta$ if $P(|\hat\theta - \theta| > \epsilon) \to$ ________, ________ > 0, **key:** $\underline{0}$, $\underline{\forall\ \epsilon}$,
9. If (1) $E(\hat\theta) =$ ____ and (2) $V(\hat\theta)$ ________, as $n \to \infty$, then $\hat\theta$ is consistent.
   key: $\underline{\theta}$, $\underline{\to 0}$,
10. If $g(\cdot)$ is ____________, $\hat\theta_1 \xrightarrow{P} \theta_1$ and $\hat\theta_2 \xrightarrow{P} \theta_2$,
    then $\hat\theta_1 \pm \hat\theta_2 \xrightarrow{P}$ ________, $\hat\theta_1\hat\theta_2 \xrightarrow{P}$ ________, $g(\hat\theta_1) \xrightarrow{P}$ ________,
    key: *cts*, $\underline{\theta_1 \pm \theta_2}$, $\underline{\theta_1\theta_2}$, $\underline{g(\theta_1)}$,
13. Given a random sample X_1, ..., X_n from $f(x;\theta)$, their likelihood is $L(\theta) =$ __________, the MLE $\hat\theta$ of θ maximizes ________. If $g(\theta)$ is a ________ function of θ, the MLE of $g(\theta)$ is ________.
    key: $\underline{\prod_i f(X_i;\theta)}$, $\underline{L(\theta)}$. $\underline{1-1}$, $\underline{g(\hat\theta)}$,
14. Given a random sample from $f(x;\theta)$, and a function $g(\theta)$, under certain conditions, $P(g(\hat\theta) \le t)$ ____ $\Phi(\frac{t-g(\theta)}{\hat\sigma_{g(\hat\theta)}})$, where $\hat\theta$ is the _____ of θ, $\hat\sigma_{g(\hat\theta)} =$ ________________
    key: $\underline{\approx}$, *MLE*, $\underline{\sqrt{\{(\frac{\partial g}{\partial\theta})^2/(-nE(\frac{\partial^2 \log f(X;\theta)}{\partial\theta^2}))\}\Big|_{\theta=\hat\theta}}}$,
15. The 5 elements of a test are (1) ________, (2) ________, (3) test statistics (4) ________, (5) ________, **key:** $\underline{H_o}$, $\underline{H_a}$, $\underline{RR}$, *Conclusion*,
16. Probability of type I error is ________, Probability of type II error is ________, **key:** $\underline{P(H_a|H_o)}$, $\underline{P(H_o|H_a)}$,
17. For a large sample test for H_o: $\theta = \theta_o$, a test statistic is $Z =$ ____________, a RR is Z________ if $\theta > \theta_o$; and a RR is ______________ if $\theta \ne \theta_o$; **key:** $\underline{\frac{\hat\theta-\theta_o}{\hat\sigma_{\hat\theta}}}$, $\underline{> z_\alpha}$, $\underline{|Z| > z_{\alpha/2}}$,
19. The P-value is $\begin{cases} P(W\_\_\_\_w)|H_o \text{ is correct}) & \text{if } H_a:\ \theta > \theta_o \\ P(W\_\_\_\_w)|H_o \text{ is correct}) & \text{if } H_a:\ \theta < \theta_o \\ \_\_\_P(W\_\_\_\_|w|)|H_o \text{ is correct}) & \text{if } H_a:\ \theta \ne \theta_o \end{cases}$ where W is the (Z or T) test statistic and w is the observed value of W. **key:** $\underline{\ge}$, $\underline{\le}$, $\underline{2}$, $\underline{\ge}$,
20. Suppose that $Z \sim N(0,1)$, $X \sim \chi^2(u)$, $Y \sim \chi^2(v)$. If Z____X, $T =$ ____________, then $T \sim t_u$; If X____Y, $F =$ ____, then $F \sim F_{u,v}$ and $X + Y \sim$ ____________. **key:** $\underline{\perp}$, $\underline{Z/\sqrt{X/u}}$, $\underline{\perp}$, $\underline{\frac{X/u}{Y/v}}$, $\underline{\chi^2(u+v)}$,
21. The MP test for H_o: $\theta = \theta_o$ v.s. H_a: $\theta = \theta_a$, the MP test has the RR satisfying: $\frac{L(\theta_o)}{L(\theta_a)}$____$k$ and $P_\theta(RR) = \alpha$ if $\theta =$ ____. **key:** $\underline{\le}$, $\underline{\theta_o}$,
22. The Likelihood ratio test for H_o: $\theta \in \Theta_o$ v.s. H_a: $\theta \notin \Theta_o$ has a RR: $\{\lambda$____$k\}$, where $\lambda =$ _________; $\hat\theta_o$ is the MLE under ____; $\hat\theta$ is the MLE under ____;
    k satisfies $max\{P(RR) :\ \theta \in \Theta_o\} =$ ____; if n is large, then $-2\ln\lambda$ is approximated _______;
    where $v =$ ___________; r and r_o = # of free parameters in Θ and in Θ_o, respectively.
    key: $\underline{\le}$, $\underline{\frac{L(\hat\theta_o)}{L(\hat\theta)}}$, $\underline{\Theta_o}$, $\underline{\Theta}$, $\underline{\alpha}$, $\underline{\chi^2(v)}$, $\underline{r - r_o}$