Appendix D

Tables

In this appendix we show tables of selected functions for the Standard Ultimate and Select Survival Models. These are used extensively throughout this book for examples and exercises.

The **Standard Ultimate Survival Model** follows Makeham's Law, parameterized as follows:

$$\mu_x = A + Bc^x \qquad \text{where } A = 0.00022,\ B = 2.7 \times 10^{-6},\ c = 1.124. \tag{D.1}$$

The **Standard Select Survival Model** is defined as follows:

- The select period is two years.
- The ultimate part of the model is the Standard Ultimate Survival Model.
- For the select part of the model, for $0 \le s \le 2$,

$$\mu_{[x]+s} = 0.9^{2-s}\mu_{x+s}. \tag{D.2}$$

This model was introduced in Example 3.13.

We also present, for convenience of access, the pension service table derived and used for examples and exercises in Chapter 10.

Note that all the solutions to the examples and exercises in the text, and in the accompanying model solutions, were calculated directly from the models, not from these tables. Using these tables will introduce rounding errors, resulting in some small differences compared with the answers in the text.

The tables are designed to provide a convenient reference for readers, and to assist with exam preparation for exam candidates. However, there are many examples and exercises in the text that cannot be answered using these tables. It is recommended that readers create their own spreadsheets, which can be used when the parameters or interest rates required are different from those used here.

Table D.1 *Standard Select and Ultimate Survival Model.*

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$	x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
			100000.00	20	60	96568.13	96287.48	95940.60	62
			99975.04	21	61	96232.34	95920.27	95534.43	63
20	99995.08	99973.75	99949.71	22	62	95858.91	95511.80	95082.53	64
21	99970.04	99948.40	99923.98	23	63	95443.51	95057.36	94579.73	65
22	99944.63	99922.65	99897.79	24	64	94981.34	94551.72	94020.33	66
23	99918.81	99896.43	99871.08	25	65	94467.11	93989.16	93398.05	67
24	99892.52	99869.70	99843.80	26	66	93895.00	93363.38	92706.06	68
25	99865.69	99842.38	99815.86	27	67	93258.63	92667.50	91936.88	69
26	99838.28	99814.41	99787.20	28	68	92551.02	91894.03	91082.43	70
27	99810.20	99785.70	99757.71	29	69	91764.58	91034.84	90133.96	71
28	99781.36	99756.17	99727.29	30	70	90891.07	90081.15	89082.09	72
29	99751.69	99725.70	99695.83	31	71	89921.62	89023.56	87916.84	73
30	99721.06	99694.18	99663.20	32	72	88846.72	87852.03	86627.64	74
31	99689.36	99661.48	99629.26	33	73	87656.25	86555.99	85203.46	75
32	99656.47	99627.47	99593.83	34	74	86339.55	85124.37	83632.89	76
33	99622.23	99591.96	99556.75	35	75	84885.49	83545.75	81904.34	77
34	99586.47	99554.78	99517.80	36	76	83282.61	81808.54	80006.23	78
35	99549.01	99515.73	99476.75	37	77	81519.30	79901.17	77927.35	79
36	99509.64	99474.56	99433.34	38	78	79584.04	77812.44	75657.16	80
37	99468.12	99431.02	99387.29	39	79	77465.70	75531.88	73186.31	81
38	99424.18	99384.82	99338.26	40	80	75153.97	73050.22	70507.19	82
39	99377.52	99335.62	99285.88	41				67614.60	83
40	99327.82	99283.06	99229.76	42				64506.50	84
41	99274.69	99226.72	99169.41	43				61184.88	85
42	99217.72	99166.14	99104.33	44				57656.68	86
43	99156.42	99100.80	99033.94	45				53934.73	87
44	99090.27	99030.10	98957.57	46				50038.65	88
45	99018.67	98953.40	98874.50	47				45995.64	89
46	98940.96	98869.96	98783.91	48				41841.05	90
47	98856.38	98778.94	98684.88	49				37618.56	91
48	98764.09	98679.44	98576.37	50				33379.88	92
49	98663.15	98570.40	98457.24	51				29183.78	93
50	98552.51	98450.67	98326.19	52				25094.33	94
51	98430.98	98318.95	98181.77	53				21178.30	95
52	98297.24	98173.79	98022.38	54				17501.76	96
53	98149.81	98013.56	97846.20	55				14125.89	97
54	97987.03	97836.44	97651.21	56				11102.53	98
55	97807.07	97640.40	97435.17	57				8469.73	99
56	97607.84	97423.18	97195.56	58				6248.17	100
57	97387.05	97182.25	96929.59	59					
58	97142.13	96914.80	96634.14	60					
59	96870.22	96617.70	96305.75	61					

Table D.2 *Standard Select Survival Model, $i = 5\%$ per year.*

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	${}_5E_{[x]}$	${}_{10}E_{[x]}$	${}_{20}E_{[x]}$	x
20	19.967	0.04918	0.00576	0.78255	0.61227	0.37441	20
21	19.921	0.05140	0.00610	0.78254	0.61223	0.37431	21
22	19.872	0.05373	0.00648	0.78252	0.61218	0.37419	22
23	19.820	0.05618	0.00689	0.78249	0.61213	0.37406	23
24	19.766	0.05874	0.00734	0.78247	0.61208	0.37392	24
25	19.710	0.06143	0.00783	0.78244	0.61201	0.37375	25
26	19.651	0.06424	0.00837	0.78241	0.61194	0.37356	26
27	19.589	0.06720	0.00895	0.78237	0.61186	0.37336	27
28	19.524	0.07029	0.00959	0.78233	0.61177	0.37312	28
29	19.456	0.07353	0.01028	0.78229	0.61167	0.37286	29
30	19.384	0.07693	0.01104	0.78224	0.61156	0.37256	30
31	19.310	0.08049	0.01186	0.78218	0.61143	0.37223	31
32	19.232	0.08421	0.01276	0.78211	0.61128	0.37186	32
33	19.150	0.08811	0.01373	0.78204	0.61112	0.37144	33
34	19.064	0.09220	0.01479	0.78196	0.61094	0.37097	34
35	18.974	0.09647	0.01594	0.78187	0.61074	0.37044	35
36	18.880	0.10094	0.01720	0.78176	0.61051	0.36985	36
37	18.782	0.10562	0.01856	0.78165	0.61025	0.36919	37
38	18.679	0.11051	0.02004	0.78152	0.60996	0.36844	38
39	18.572	0.11563	0.02164	0.78137	0.60963	0.36761	39
40	18.460	0.12097	0.02338	0.78121	0.60927	0.36667	40
41	18.342	0.12656	0.02527	0.78102	0.60886	0.36562	41
42	18.220	0.13240	0.02731	0.78082	0.60840	0.36444	42
43	18.092	0.13849	0.02952	0.78058	0.60788	0.36312	43
44	17.958	0.14485	0.03191	0.78032	0.60730	0.36165	44
45	17.819	0.15149	0.03450	0.78003	0.60664	0.35999	45
46	17.673	0.15841	0.03730	0.77970	0.60591	0.35815	46
47	17.522	0.16563	0.04032	0.77932	0.60509	0.35608	47
48	17.364	0.17314	0.04358	0.77891	0.60416	0.35377	48
49	17.200	0.18098	0.04709	0.77844	0.60313	0.35120	49

Table D.2 (*Cont.*)

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	${}_5E_{[x]}$	${}_{10}E_{[x]}$	${}_{20}E_{[x]}$	x
50	17.028	0.18913	0.05087	0.77791	0.60196	0.34832	50
51	16.850	0.19761	0.05495	0.77732	0.60066	0.34512	51
52	16.665	0.20642	0.05933	0.77665	0.59919	0.34156	52
53	16.473	0.21558	0.06404	0.77591	0.59755	0.33760	53
54	16.273	0.22509	0.06909	0.77507	0.59572	0.33320	54
55	16.066	0.23496	0.07451	0.77413	0.59366	0.32832	55
56	15.851	0.24519	0.08031	0.77307	0.59135	0.32293	56
57	15.628	0.25579	0.08653	0.77189	0.58877	0.31697	57
58	15.398	0.26677	0.09317	0.77056	0.58588	0.31041	58
59	15.160	0.27811	0.10025	0.76907	0.58265	0.30319	59
60	14.913	0.28984	0.10781	0.76739	0.57904	0.29528	60
61	14.659	0.30194	0.11586	0.76552	0.57501	0.28663	61
62	14.397	0.31442	0.12441	0.76341	0.57051	0.27721	62
63	14.127	0.32727	0.13350	0.76105	0.56550	0.26700	63
64	13.850	0.34049	0.14313	0.75841	0.55992	0.25596	64
65	13.564	0.35407	0.15333	0.75545	0.55371	0.24411	65
66	13.272	0.36801	0.16411	0.75214	0.54682	0.23143	66
67	12.972	0.38230	0.17548	0.74844	0.53917	0.21797	67
68	12.665	0.39692	0.18746	0.74429	0.53070	0.20377	68
69	12.351	0.41186	0.20005	0.73966	0.52134	0.18891	69
70	12.031	0.42710	0.21326	0.73450	0.51102	0.17350	70
71	11.705	0.44262	0.22709	0.72873	0.49966	0.15767	71
72	11.374	0.45840	0.24154	0.72230	0.48719	0.14160	72
73	11.037	0.47442	0.25662	0.71515	0.47355	0.12548	73
74	10.696	0.49065	0.27229	0.70719	0.45867	0.10954	74
75	10.352	0.50706	0.28856	0.69835	0.44250	0.09403	75
76	10.004	0.52362	0.30541	0.68854	0.42501	0.07920	76
77	9.654	0.54029	0.32280	0.67768	0.40618	0.06531	77
78	9.302	0.55704	0.34072	0.66568	0.38600	0.05258	78
79	8.950	0.57382	0.35912	0.65245	0.36451	0.04121	79
80	8.597	0.59061	0.37797	0.63789	0.34179	0.03133	80

Table D.3 *Standard Ultimate Survival Model, $i = 5\%$ per year.*

x	$\ddot{a}_x$	A_x	${}^2A_x$	${}_5E_x$	${}_{10}E_x$	${}_{20}E_x$	x
20	19.966	0.04922	0.00580	0.78252	0.61224	0.37440	20
21	19.920	0.05144	0.00614	0.78250	0.61220	0.37429	21
22	19.871	0.05378	0.00652	0.78248	0.61215	0.37417	22
23	19.819	0.05622	0.00694	0.78245	0.61210	0.37404	23
24	19.765	0.05879	0.00739	0.78243	0.61205	0.37390	24
25	19.709	0.06147	0.00788	0.78240	0.61198	0.37373	25
26	19.650	0.06429	0.00841	0.78236	0.61191	0.37354	26
27	19.588	0.06725	0.00900	0.78233	0.61183	0.37334	27
28	19.523	0.07034	0.00964	0.78229	0.61174	0.37310	28
29	19.455	0.07359	0.01033	0.78224	0.61163	0.37284	29
30	19.383	0.07698	0.01109	0.78219	0.61152	0.37254	30
31	19.309	0.08054	0.01192	0.78213	0.61139	0.37221	31
32	19.230	0.08427	0.01281	0.78206	0.61124	0.37183	32
33	19.148	0.08817	0.01379	0.78199	0.61108	0.37141	33
34	19.063	0.09226	0.01486	0.78190	0.61090	0.37094	34
35	18.973	0.09653	0.01601	0.78181	0.61069	0.37041	35
36	18.879	0.10101	0.01727	0.78170	0.61046	0.36982	36
37	18.780	0.10569	0.01863	0.78158	0.61020	0.36915	37
38	18.678	0.11059	0.02012	0.78145	0.60990	0.36841	38
39	18.570	0.11571	0.02173	0.78130	0.60957	0.36757	39
40	18.458	0.12106	0.02347	0.78113	0.60920	0.36663	40
41	18.340	0.12665	0.02536	0.78094	0.60879	0.36558	41
42	18.218	0.13249	0.02741	0.78072	0.60832	0.36440	42
43	18.090	0.13859	0.02963	0.78048	0.60780	0.36307	43
44	17.956	0.14496	0.03203	0.78021	0.60721	0.36159	44
45	17.816	0.15161	0.03463	0.77991	0.60655	0.35994	45
46	17.671	0.15854	0.03744	0.77956	0.60581	0.35809	46
47	17.519	0.16577	0.04047	0.77918	0.60498	0.35601	47
48	17.361	0.17330	0.04374	0.77875	0.60404	0.35370	48
49	17.196	0.18114	0.04727	0.77827	0.60299	0.35112	49

Table D.3 (*Cont.*)

x	$\ddot{a}_x$	A_x	${}^2A_x$	${}_5E_x$	${}_{10}E_x$	${}_{20}E_x$	x
50	17.025	0.18931	0.05108	0.77772	0.60182	0.34824	50
51	16.846	0.19780	0.05517	0.77711	0.60050	0.34503	51
52	16.661	0.20664	0.05957	0.77643	0.59902	0.34146	52
53	16.468	0.21582	0.06430	0.77566	0.59736	0.33749	53
54	16.268	0.22535	0.06938	0.77479	0.59550	0.33308	54
55	16.060	0.23524	0.07483	0.77382	0.59342	0.32819	55
56	15.844	0.24550	0.08067	0.77273	0.59109	0.32279	56
57	15.621	0.25613	0.08692	0.77151	0.58848	0.31681	57
58	15.390	0.26714	0.09360	0.77014	0.58556	0.31024	58
59	15.151	0.27852	0.10073	0.76860	0.58229	0.30300	59
60	14.904	0.29028	0.10834	0.76687	0.57864	0.29508	60
61	14.649	0.30243	0.11644	0.76493	0.57457	0.28641	61
62	14.386	0.31495	0.12506	0.76276	0.57003	0.27698	62
63	14.115	0.32785	0.13421	0.76033	0.56496	0.26674	63
64	13.836	0.34113	0.14392	0.75760	0.55932	0.25569	64
65	13.550	0.35477	0.15420	0.75455	0.55305	0.24381	65
66	13.256	0.36878	0.16507	0.75114	0.54609	0.23112	66
67	12.954	0.38313	0.17654	0.74732	0.53836	0.21764	67
68	12.646	0.39783	0.18862	0.74305	0.52981	0.20343	68
69	12.330	0.41285	0.20133	0.73828	0.52036	0.18856	69
70	12.008	0.42818	0.21467	0.73295	0.50994	0.17313	70
71	11.680	0.44379	0.22864	0.72701	0.49848	0.15730	71
72	11.347	0.45968	0.24324	0.72039	0.48590	0.14122	72
73	11.008	0.47580	0.25847	0.71303	0.47215	0.12511	73
74	10.665	0.49215	0.27433	0.70483	0.45715	0.10918	74
75	10.318	0.50868	0.29079	0.69574	0.44085	0.09368	75
76	9.967	0.52536	0.30783	0.68566	0.42323	0.07887	76
77	9.614	0.54217	0.32544	0.67450	0.40427	0.06500	77
78	9.260	0.55906	0.34359	0.66217	0.38396	0.05230	78
79	8.904	0.57599	0.36224	0.64859	0.36235	0.04096	79
80	8.548	0.59293	0.38134	0.63365	0.33952	0.03113	80

Table D.4 *Pension plan service table.*

x	l_x	w_x	i_x	r_x	d_x	x	l_x	w_x	i_x	r_x	d_x
20	1 000 000	95 104	951	0	237	44	137 656	6 708	134	0	95
21	903 707	85 946	859	0	218	45	130 719	2 586	129	0	100
22	816 684	77 670	777	0	200	46	127 904	2 530	127	0	106
23	738 038	70 190	702	0	184	47	125 140	2 476	124	0	113
24	666 962	63 430	634	0	170	48	122 428	2 422	121	0	121
25	602 728	57 321	573	0	157	49	119 763	2 369	118	0	130
26	544 677	51 800	518	0	145	50	117 145	2 317	116	0	140
27	492 213	46 811	468	0	134	51	114 572	2 266	113	0	151
28	444 800	42 301	423	0	125	52	112 042	2 216	111	0	163
29	401 951	38 226	382	0	117	53	109 553	2 166	108	0	176
30	363 226	34 543	345	0	109	54	107 102	2 118	106	0	190
31	328 228	31 215	312	0	102	55	104 688	2 070	103	0	206
32	296 599	28 207	282	0	96	56	102 308	2 023	101	0	224
33	268 014	25 488	255	0	91	57	99 960	1 976	99	0	243
34	242 181	23 031	230	0	86	58	97 642	1 930	96	0	264
35	218 834	10 665	213	0	83	59	95 351	1 884	94	0	288
36	207 872	10 131	203	0	84	60^-	93 085	0	0	27 926	0
37	197 455	9 623	192	0	84	60^+	65 160	0	62	6 188	210
38	187 555	9 141	183	0	85	61	58 700	0	56	5 573	212
39	178 147	8 682	174	0	86	62	52 860	0	50	5 018	213
40	169 206	8 246	165	0	87	63	47 579	0	45	4 515	214
41	160 708	7 832	157	0	89	64	42 805	0	41	4 061	215
42	152 631	7 438	149	0	90	65^-	38 488	0	0	38 488	0
43	144 954	7 064	141	0	93						

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