

## Assignments for the week Oct. 3<sup>rd</sup> -- 10<sup>th</sup>

1. a) Find  $d = \gcd(136, 221)$  with Euclid's algorithm.  
b) Find  $x, y \in \mathbb{Z}$  with  $d = 136x + 221y$ .
2. a) Find the elements of the multiplicative group  $\mathbb{Z}_{24}^* = \{[a] \in \mathbb{Z}_{24} \mid [a] \text{ is invertible in } \mathbb{Z}_{24}\}$ .  
b) Find the order of the multiplicative group  $\mathbb{Z}_q^*$  when  $q = p^m$  is a prime power.
3. a) Is it the case that given any two given  $x, y$  elements of the dihedral group  $D_5$ , with  $x \neq e \neq y$ , then  $\text{gp}(x, y) = D_5$  (proof or counter example).  
b) Same question for  $D_6$ .
4. Let  $U, V$  be two subgroups of  $G$ . We define the *product* of  $U$  and  $V$  to be the subset  $UV = \{uv \mid u \in U, v \in V\}$ .  
  
a) Show that  $UV$  is not, in general, a subgroup, by taking the example  $G = D_5$ , and  $U, V$  cyclic generated by reflections.  
  
b) Prove that if  $UV$  is a subgroup then  $UV = VU$ .