Assignment for the week Sept. 12th – Sept 19th.

1. In the *"Modern Algebra"* notes by Matthew Brin the group of all permutations of the finite set X = {1,2,...,n } is denoted S_X or S_n and called *the symmetric group (of degree n)*.
   Read Section 2.3.1 (p. 61-63), where you find the notation for permutations we have introduced in class. The important result that every permutation has an (essentially unique) description as a product of *"disjoint cicles"* (together with the notion of the *support* of a permutation and others facts covered in class) are in Section 4.3.4 (p. 135-139).
   Read Section 4.3.4 carefully and do ask in class when you find something which you do not fully understand.

2. **Write** all 24 permutations of S_n as products of disjoint cycles.

3. **Find and read** the definition of a *"group"* the Text.

4. **Read, discuss, understand (or ask!) and solve the following problems.**

a) Assume every element $a \in G$ has Type name of new folderthe property that $a^2 = e$. Prove that G is Abelian.

b) How many (essentially different) groups G of order $|G| = 1, 2, 3, 4$ do you find? Write down their *multiplication table* – it exhibits for every pair $(a,b) \in G \times G$ the product$ab \in G$.
("essentially different" here means: if renaming the elements of a group G leads to a multiplication table of a second group H then G and H should be considered "the same group").

c) Prove that $SL_2(\mathbb{Z})$, the set of all integral 2×2-matrices A with $\det A = 1$, is a group.

d) Let a be an element of a group G. Prove that the maps $f, g: G \to G$ defined by $f(x) = ax$, $g(x) = axa^{-1}$ are bijections.

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