Assignment for the week Sept. 5th – Sept 12th.

1. Read Chapter 2.2 (sets and functions, p. 50 – 59) carefully. Ask questions in class when you find something which you do not fully understand.

2. Do exercises 15, 16, 17.

3. Try to read, discuss, understand and solve the following set of problems.

When A is a set we write P(A) for the set of all subsets of A; thus $P(A) = \{X \mid X \subseteq A\}$.
When $f: A \to B$ is a function from the set A into the set B, and $S \subseteq B$ is an arbitrary subset of B we write $f^{-1}(S)$ for the subset of A, $f^{-1}(S) = \{a \in A \mid f(a) \in S\}$. Thus f^{-1} assigns to every subset S of B a subset $f^{-1}(S)$ of A; hence f^{-1} is a function $f^{-1}: P(B) \to P(A)$. – See p. 57 for more details.

a) Find $f^{-1}(S)$ for $S = \varnothing$ (the empty set), for $S = B$, and describe $f^{-1}(S)$ in words when when $S = \{b\}$ is a singleton.

b) Is it true that for all S and T in P(B) we have

$$f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T) \text{ , and}$$

$$f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T) \text{ ?}$$

c) Show that if b is an element of B which is not in f(A) then $f^{-1}(\{b\}) = \varnothing$
c') Prove that if f^{-1} is injective then f is surjective.

d) Show that if a and a ' are two elements of A with $f(a) = f(a')$ then every set of the form $f^{-1}(S)$ (with $S \subseteq B$) either contains both a and a' or contains neither a nor a'.
d') Prove that if f^{-1} is surjective then f is injective.