Test 1

September 27^{th} 2012

1. Let $A = (\mathbb{Z}_8,+)$ be the additive group of the integers mod 8. Find the order of all elements in A.

2. Let M be the set of all elements of $\mathbb{Z}_8$ that have a multiplicative inverse in $\mathbb{Z}_8$. Prove that M -- with respect to multiplication in $\mathbb{Z}_8$ -- is a group, and compute the order of its elements.

3. Let G be an arbitrary group, and let $t \in G$. Prove that for each $x \in G$, x and $y = t^{-1}xt$ have the same order.

4. Let $f: A \rightarrow B$ be a function from a set A into a set B, and $g: P(B) \rightarrow P(A)$ the function on set of subsets which sends the subset $Y \subseteq B$ to the subset $g(Y) = \{a \in A \mid f(a) \in Y\} \subseteq A$
   Show that if f is surjective then g is injectiv.

5. Consider the two permutations $\pi = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 3\,4\,6\,2\,1\,7\,5 \end{pmatrix}$ $\rho = \begin{pmatrix} 12\,3\,4\,5\,6\,7 \\ 2\,17\,5\,6\,4\;\;3 \end{pmatrix}$.
   Find a permutation τ which has the property that $\tau\pi = \rho$.

6. Compute the orders of the elements τ, π, and ρ in Problem 5.

7. Give the Definition of a group $(G,*)$ in detail (be careful to use the words *"for all ..."* and *"there is ..."* correctly!)

8. In the dihedral group D_4 let ρ be the rotation by $+\pi/2$. Choose an arbitrary reflection σ and compute $\sigma\rho\sigma^{-1}$.

9. Let A be a set with 3 elements, and let $G = P(A)$ be the power set of A. We know that G with the operation $X*Y = \{a \in A \mid a \in X \cup Y, a \notin X \cap Y\}$, where $X,Y \in A$, is a group. Give the multiplication table of G and decide whether G is isomorphic to $\mathbb{Z}_8$.