

Typical Test Type Questions

1. Find the order of the permutation $\pi =$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 3 & 6 & 2 & 1 \end{pmatrix}$$

2. Find an element of maximal order in the symmetric permutation group S_5 .
3. Write down the multiplication table of the group $(\mathbb{Z}_4, +)$.
4. In the dihedral group D_4 – the symmetry group of the square Q -- we write ρ for the rotation by $+\pi/2$ an σ , σ' the reflection whose axis go through two opposite corners of Q , and τ , τ' the reflections over axes through midpoints of edges of Q .
- a) compute the products $\sigma\sigma'$, $\tau\tau'$, $\tau\sigma'$, $\tau'\sigma$.
- b) Find an element $\alpha \in D_4$ with the property that the composition $\sigma\alpha$ is the reflection over an axis through edge midpoints of Q .
5. In D_n find all pairs of commuting reflections.
6. Prove that the multiplication in \mathbb{Z}_m defined by $[a][b]=[ab]$ is well defined.
7. Let $f: \mathbb{Z}_m \rightarrow \mathbb{Z}_m$ the function given by $f([a])=[a^2]$. a) Prove that f is well defined, and b) Find the image $f(\mathbb{Z}_m)$ when $m=4$.
8. Let X be a finite set and S_X the group of all permutations of X . We consider the function $f: S_X \rightarrow \mathcal{P}(X)$ the function which assigns to each permutation π the support $\text{supp}(\pi)$.
- a) For every subset $A \subseteq X$ describe the fiber $f(A)$ in words, and
- b) compute $|f(A)|$ -- the number of elements in the fiber of A -- in terms of $|A|$.